opHW9

2019年3月25日 10:01

Sec4.1, Optional ones: 2*, 14*, 19*(b) (find kernel only), 22*

来自 <https://users.math.msu.edu/users/zhangshiwen/s19/homework.html>

2. Let L be the linear operator on $\mathbb{R}^2$ defined by

$$L(\mathbf{x}) = (x_1 \cos\alpha - x_2 \sin\alpha,\ x_1 \sin\alpha + x_2 \cos\alpha)^T$$

Express x_1, x_2, and $L(\mathbf{x})$ in terms of polar coordinates. Describe geometrically the effect of the linear transformation.

polar coordinates $(r, \theta) \longrightarrow (x_1, x_2)$

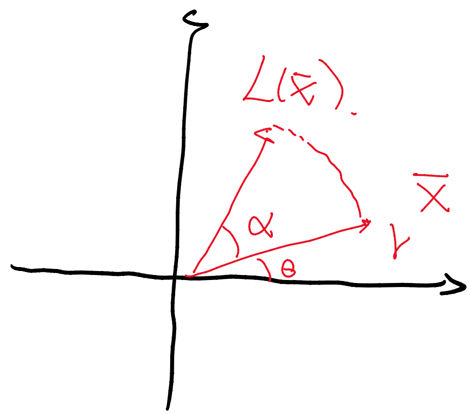
$$\begin{cases} x_1 = r\cdot\cos\theta \\ x_2 = r\cdot\sin\theta \end{cases}$$

$L(\vec{x})$:

$$x_1\cdot\cos\alpha - x_2\cdot\sin\alpha = r\cdot\cos\theta\cos\alpha - r\cdot\sin\theta\sin\alpha$$
$$= r\cdot\cos(\theta+\alpha)$$

$$x_1\cdot\sin\alpha + x_2\cdot\cos\alpha = r\cdot\cos\theta\cdot\sin\alpha + r\cdot\sin\theta\cdot\cos\alpha$$
$$= r\cdot\sin(\theta+\alpha).$$

The linear transformation rotates the vector $\vec{x}$ by the angle α counterclockwise



14. Let L be a linear operator on $\mathbb{R}^1$ and let $a = L(1)$. Show that $L(x) = ax$ for all $x \in \mathbb{R}^1$.

$$L(x) = L(x \cdot 1) = x L(1)$$
$$= x \cdot a.$$

19. Find the kernel and ~~range~~ of each of the following linear operators on P_3:

(a) $L(p(x)) = xp'(x)$ (b) $L(p(x)) = p(x) - p'(x)$

(c) $L(p(x)) = p(0)x + p(1)$

(b). $p(x) = a_0 + a_1 x + a_2 x^2$

$p'(x) = a_1 + 2a_2 x$

$L(p) = a_0 + a_1 x + a_2 x^2 - (a_1 + 2a_2 x)$

$= a_0 - a_1 + (a_1 - 2a_2)x + a_2 x^2$

Let $L(p(x)) = 0$

Let $L(p(x)) = 0$

$a_0 - a_1 = 0, \quad a_1 - 2a_2 = 0, \quad a_2 = 0$

$\Rightarrow a_0 = a_1 = a_2 = 0 \Rightarrow p(x) = 0$

$\ker(L) = \{0\}$.

Sec 4.2

5. Find the standard matrix representation for each of the following linear operators:

(a) L is the linear operator that rotates each $\mathbf{x}$ in $\mathbb{R}^2$ by 45° in the clockwise direction.

(b) L is the linear operator that reflects each vector $\mathbf{x}$ in $\mathbb{R}^2$ about the x_1 axis and then rotates it 90° in the counterclockwise direction.

(c) L doubles the length of $\mathbf{x}$ and then rotates it 30° in the counterclockwise direction.

(d) L reflects each vector $\mathbf{x}$ about the line $x_2 = x_1$ and then projects it onto the x_1-axis.

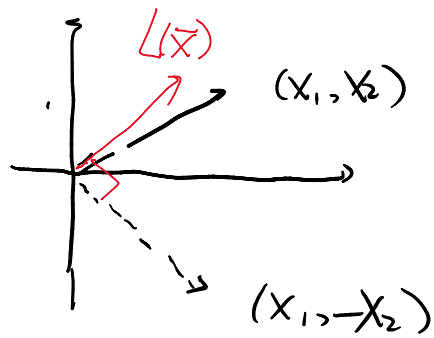
(a). By #2, $\cos(-45°) = \frac{\sqrt{2}}{2}$, $\sin(-45°) = -\frac{\sqrt{2}}{2}$

$L(\mathbf{x}) = \left(x_1 \cdot \frac{\sqrt{2}}{2} - x_2(-\frac{\sqrt{2}}{2}), \quad x_1(-\frac{\sqrt{2}}{2}) + x_2 \cdot \frac{\sqrt{2}}{2}\right)^T$

$= \begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ \sqrt{2} & \sqrt{} \end{bmatrix} \begin{bmatrix} x_1 \\ \end{bmatrix}$

$$= \underbrace{\begin{bmatrix} & \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix}}_{\text{Matrix representation.}} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

(b)

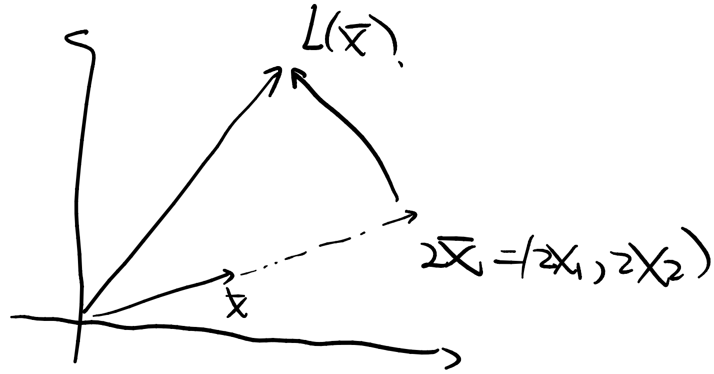


$$L(\bar{x}) = \left(x_1 \cdot \cos 90^\circ - (-x_2)\sin 90^\circ,\ x_1 \sin 90^\circ + (-x_2)\cos 90^\circ\right)^T$$

$$= (x_2, x_1)^T$$

$$= \underbrace{\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}}_{\text{Matrix representation}} \begin{bmatrix} x_2 \\ x_1 \end{bmatrix}$$

(c)

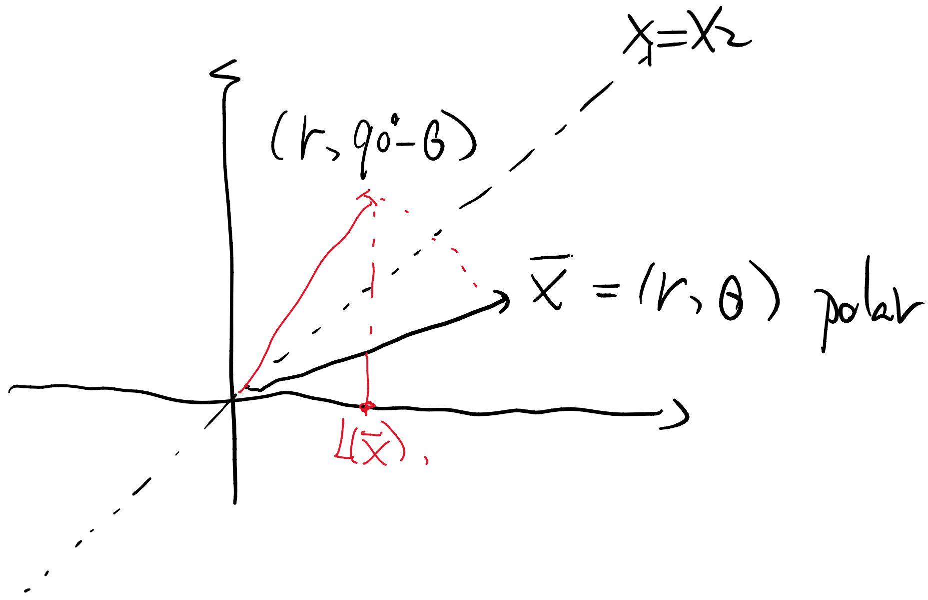


$$L(\bar{x}) = \left(2x_1 \cdot \cos 30^\circ - 2x_2 \sin 30^\circ,\ 2x_1 \sin 30^\circ + 2x_2 \cdot \cos 30^\circ\right)^T$$

$$= \left(2x_1 \cdot \frac{\sqrt{3}}{2} - 2x_2 \cdot \frac{1}{2},\ 2x_1 \cdot \frac{1}{2} + 2x_2 \cdot \frac{\sqrt{3}}{2}\right)^T$$

$$= \underbrace{\begin{bmatrix} \sqrt{3} & -1 \\ 1 & \sqrt{3} \end{bmatrix}}_{\text{Matrix repkes.}} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}.$$

(d)



$$\begin{cases} x_1 = r \cdot \cos\theta \\ x_2 = r \cdot \sin\theta. \end{cases}$$

$$r \cdot \cos(90^\circ - \theta) = r \cdot \sin\theta = x_2.$$

$$r \cdot \cos(90^\circ - \theta) = r \cdot \sin\theta = x_2.$$

$$L(\bar{x}) = (x_2, 0).$$