opHW6

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Sec 3.2 Optional ones: 6* (c)(d), 12*(a)(b), 14*, 19*

来自 <https://users.math.msu.edu/users/zhangshiwen/s19/homework.html>

6. Determine whether the following are subspaces of $C[-1, 1]$:

(a) The set of functions f in $C[-1, 1]$ such that $f(-1) = f(1)$

(b) The set of odd functions in $C[-1, 1]$

(c) The set of continuous nondecreasing functions on $[-1, 1]$

(d) The set of functions f in $C[-1, 1]$ such that $f(-1) = 0$ and $f(1) = 0$

(c). No

$f(x) = x$ is a continuous nondecreasing function on $[-1, 1]$

For $\alpha = -1$.

$\alpha \cdot f(x) = -x$ is decreasing.

(d). Yes.

Let $f(x), g(x) \in C[-1, 1]$ and

$f(-1) = f(1) = g(-1) = g(1) = 0$.

Let $h(x) = f(x) + g(x)$

Let $h(x) = f(x) + g(x)$

Then $h(1) = h(-1) = 0$

and $S(x) = \alpha \cdot f(x)$ for any α

$S(-1) = S(1) = 0$

Therefore, it is a subspace.

12. Which of the sets that follow are spanning sets for $\mathbb{R}^3$? Justify your answers.

(a) $\{(1,0,0)^T, (0,1,1)^T, (1,0,1)^T\}$

(b) $\{(1,0,0)^T, (0,1,1)^T, (1,0,1)^T, (1,2,3)^T\}$

(c) $\{(2,1,-2)^T, (3,2,-2)^T, (2,2,0)^T\}$

(d) $\{(2,1,-2)^T, (-2,-1,2)^T, (4,2,-4)^T\}$

(e) $\{(1,1,3)^T, (0,2,1)^T\}$

(a) Let $X = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}$

$\det X = 1 \neq 0$,

Therefore, the three vectors span $\mathbb{R}^3$

(b). Consider the linear system:

$$c_1 \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + c_2 \cdot \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} + c_3 \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + c_4 \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

$$c_1\begin{pmatrix}1\\0\\0\end{pmatrix}+c_2\begin{pmatrix}0\\1\\1\end{pmatrix}+c_3\begin{pmatrix}1\\0\\1\end{pmatrix}+c_4\begin{pmatrix}1\\2\\3\end{pmatrix}=\begin{pmatrix}b_1\\b_2\\b_3\end{pmatrix}$$

$$\Leftrightarrow \begin{bmatrix}1 & 0 & 1 & 1\\ 0 & 1 & 0 & 2\\ 0 & 1 & 1 & 3\end{bmatrix}\cdot\begin{pmatrix}c_1\\c_2\\c_3\\c_4\end{pmatrix}=\begin{pmatrix}b_1\\b_2\\b_3\end{pmatrix}$$

The augmented matrix,

$$\left[\begin{array}{cccc|c}1 & 0 & 1 & 1 & b_1\\ 0 & 1 & 0 & 2 & b_2\\ 0 & 1 & 1 & 3 & b_3\end{array}\right].$$

$$\to \left[\begin{array}{cccc|c}1 & 0 & 0 & -2 & b_1-b_3\\ 0 & 0 & 0 & 2 & b_2-b_3\\ 0 & 1 & 1 & 3 & b_3\end{array}\right].$$

The system has two free variables and is consistent for any b_1, b_2, b_3

Therefore, the 4 vectors span $\mathbb{R}^3$.

Alternative proof:

By (a), the first three vectors span $\mathbb{R}^3$.

therefore $\begin{pmatrix}1\\ \vdots\end{pmatrix}$, ...

therefore, $\begin{pmatrix}1\\2\\3\end{pmatrix}$ can be written as

a linear combination of the first 3.

Therefore, the four vectors also span $\mathbb{R}^3$.

14. Let A be a 4×3 matrix and let $\mathbf{b} \in \mathbb{R}^4$. How many possible solutions could the system $A\mathbf{x} = \mathbf{b}$ have if $N(A) = \{\mathbf{0}\}$? Answer the same question in the case $N(A) \neq \{\mathbf{0}\}$. Explain your answers.

If $N(A) = \{\bar{0}\}$, then the reduced row echelon of A should be

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}.$$

Otherwise, $A\bar{x} = \bar{0}$ would have free variables.

Therefore, $A\bar{x} = \bar{b}$ either has no solution or a unique solution.

If $N(A) \neq \{0\}$,

then either $A\bar{x} = \bar{b}$ has no solution or infinitely many solutions.

19. Which of the sets that follow are spanning sets for P_3? Justify your answers.

(a) $\{1, x^2, x^2 - 2\}$ **(b)** $\{2, x^2, x, 2x + 3\}$

(c) $\{x + 2, x + 1, x^2 - 1\}$ **(d)** $\{x + 2, x^2 - 1\}$

(a) Not a spanning set since

$2 \cdot 1 + (-1) \cdot x^2 + (x^2 - 2) = 0$

the three vectors are linearly dependent.

(b).

$c_1 \cdot 2 + c_2 x^2 + c_3 \cdot x = 0$

$\Rightarrow c_1 = c_2 = c_3 = 0.$

$2, x^2, x$ are linearly independent

and any $p(x) = a_0 + a_1 x + a_2 x^2$ is a linear combination of these three

Therefore, $\{2, x^2, x, 2x+3\}$ span P_3.

(c). $c_1(x+2) + c_2(x+1) + c_3(x^2-1) = a_0 + a_1 x + a_2 x^2$

(c). $c_1(x+2) + c_2(x+1) + c_3(x^2-1) = a_0 + a_1x + a_2x^2$

$$\Leftrightarrow \begin{cases} 2c_1 + c_2 - c_3 = a_0 \\ c_1 + c_2 = a_1 \\ c_3 = a_2 . \end{cases}$$

The coefficient matrix.

$$A = \begin{bmatrix} 2 & 1 & -1 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$ is nonsingular since $\det A = 1 \neq 0$

Therefore, the system has a solution

The vectors span P_3.

(d). Consider

$$c_1(x+2) + c_2(x^2-1) = a_0 + a_1x + a_2x^2$$

$$\Rightarrow \begin{cases} 2c_1 - c_2 = a_0 \\ c_1 = a_1 \\ c_2 = a_2 \end{cases}$$

$$\Leftrightarrow \begin{bmatrix} 2 & -1 & \vdots & a_0 \\ 1 & 0 & \vdots & a_1 \end{bmatrix}$$

$$\Longleftrightarrow \quad \left[\begin{array}{cc|c} 2 & -1 & a_0 \\ 1 & 0 & a_1 \\ 0 & 1 & a_2 \end{array}\right] \text{ as the augmented matrix.}$$

$$\longrightarrow \left[\begin{array}{cc|c} 1 & 0 & a_1 \\ 0 & 1 & a_2 \\ 0 & -1 & a_0-2a_1 \end{array}\right]$$

$$\longrightarrow \left[\begin{array}{cc|c} 1 & 0 & a_1 \\ 0 & 1 & a_2 \\ 0 & 0 & a_0-2a_1+a_2 \end{array}\right]$$

If $a_0-2a_1+a_2 \neq 0$, i.e., $p(x)=1+x$ with $a_0=a_1=1, a_2=0$, then the system does not have a solution.

Therefore, $\{x+2, x^2-1\}$ does not span P_3