opHW4-1

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Sec 1.4, Optional ones: 16*, 20*, 21*.

来自 <https://users.math.msu.edu/users/zhangshiwen/s19/homework.html>

16. Prove that if A is nonsingular then A^T is nonsingular and

$$(A^T)^{-1} = (A^{-1})^T$$

Hint: $(AB)^T = B^T A^T$.

Proof. $(A^{-1})^T \cdot A^T = (A \cdot A^{-1})^T$

$= I^T = I$

$A^T \cdot (A^{-1})^T = (A^{-1} \cdot A)^T = I^T = I.$

Therefore, A^T is invertible and

$(A^T)^{-1} = (A^{-1})^T$.

20. Let A be an $n \times n$ matrix. Show that if $A^{k+1} = O$, then $I - A$ is nonsingular and

$$(I - A)^{-1} = I + A + A^2 + \cdots + A^k$$

Proof. $(I-A)\cdot(I+A+\cdots+A^k)$

$= I + A + \cdots + A^k - A(I+A+\cdots+A^k)$

$$= I + \underbrace{A + \cdots + A^k} - (\underbrace{A + A^2 + \cdots + A^k} + A^{k+1})$$

$$= I - A^{k+1} = I \quad \text{since } A^{k+1} = 0.$$

Similarly,

$$(I + A + \cdots + A^k)(I - A)$$

$$= I + A + \cdots + A^k - (I + \cdots + A^k)A$$

$$= I - A^{k+1} = I$$

Therefore, $(I-A)^{-1} = I + A + \cdots + A^k$

21. Given

$$R = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

show that R is nonsingular and $R^{-1} = R^T$.

Proof. $R^T = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$.

$$R \cdot R^T = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \cdot \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2\theta + \sin^2\theta & \cos\theta\cdot\sin\theta - \sin\theta\cos\theta \\ \sin\theta\cos\theta - \cos\theta\cdot\sin\theta & \sin^2\theta + \cos^2\theta \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

Similarly,

$$R^T \cdot R = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \cdot \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

Therefore, $R^{-1} = R$.