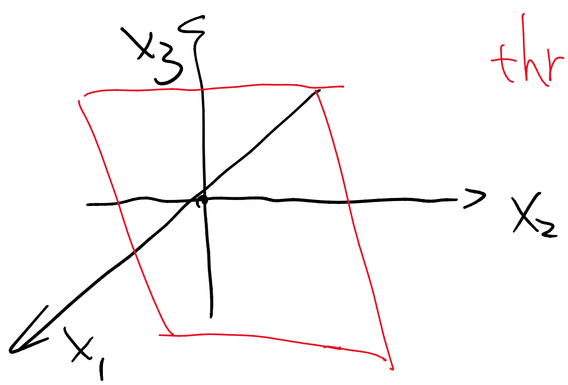
Sec1.2Optional

2019年2月3日 17:36

7. Give a geometric explanation of why a homogeneous linear system consisting of two equations in three unknowns must have infinitely many solutions. What are the possible numbers of solutions of a nonhomogeneous 2×3 linear system? Give a geometric explanation of your answer.

A linear equation with 3 unknowns represent a plane in $\mathbb{R}^3$ (x_1, x_2, x_3)

e.g. $x_1 + x_2 + x_3 = 0$ is a plane passing through the origin.



In a homogeneous system as

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = 0 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = 0 \end{cases},$$

the two planes intersect at a line in $\mathbb{R}^3$ containing the origin. In other words, In other words, all the points (x_1, x_2, x_3) on the intersection line are solutions to the system. Therefore, there are infinitely many.

8. Consider a linear system whose augmented matrix is of the form

$$\left[\begin{array}{rrr|r} 1 & 2 & 1 & 1 \\ -1 & 4 & 3 & 2 \\ 2 & -2 & a & 3 \end{array}\right]$$

For what values of a will the system have a unique solution?

Do elementary row operation

$$\left[\begin{array}{rrr|r} 1 & 2 & 1 & 1 \\ -1 & 4 & 3 & 2 \\ 2 & -2 & a & 3 \end{array}\right] \longrightarrow \left[\begin{array}{rrr|r} 1 & 2 & 1 & 1 \\ 0 & 6 & 4 & 3 \\ 0 & -6 & a-2 & 1 \end{array}\right]$$

$$\longrightarrow \left[\begin{array}{rrr|r} 1 & 2 & 1 & 1 \\ 0 & 6 & 4 & 3 \\ 0 & 0 & a+2 & 4 \end{array}\right]$$

Notice that the right hand side of the last equation is 4 (nonzero).

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If $a+2=0$, then the system is inconsistent (no solution).

As long as $\boxed{a+2 \neq 0}$, we have the last equation

$$(a+2)x_3 = 4,$$ which implies

$$\boxed{x_3 = \frac{4}{a+2}}.$$

Then we can find x_1, x_2 by back substitution.

This gives us the unique solution to the system.