Sec 1.1. Optional Homework.

7. The two systems

$$\begin{aligned} 2x_1 + x_2 &= 3 \\ 4x_1 + 3x_2 &= 5 \end{aligned} \quad \text{and} \quad \begin{aligned} 2x_1 + x_2 &= -1 \\ 4x_1 + 3x_2 &= 1 \end{aligned}$$

have the same coefficient matrix but different right-hand sides. Solve both systems simultaneously by eliminating the first entry in the second row of the augmented matrix

$$\left[\begin{array}{cc|cc} 2 & 1 & 3 & -1 \\ 4 & 3 & 5 & 1 \end{array}\right]$$

and then performing back substitutions for each of the columns corresponding to the right-hand sides.

Elementary Row operation:

$$\left[\begin{array}{cc|cc} 2 & 1 & 3 & -1 \\ 4 & 3 & 5 & 1 \end{array}\right] \xrightarrow{② - 2①} \left[\begin{array}{cc|cc} 2 & 1 & 3 & -1 \\ 0 & 1 & -1 & 3 \end{array}\right]$$

$$\xrightarrow{① - ②} \left[\begin{array}{cc|cc} 2 & 0 & 4 & -4 \\ 0 & 1 & -1 & 3 \end{array}\right] \xrightarrow{①/2} \left[\begin{array}{cc|cc} 1 & 0 & 2 & -2 \\ 0 & 1 & -1 & 3 \end{array}\right]$$

The Two Linear Systems are:

$$\begin{cases} x_1 + 0 = 2 \\ 0 + x_2 = -1 \end{cases} \quad \text{and} \quad \begin{cases} x_1 + 0 = -2 \\ 0 + x_2 = 3 \end{cases}$$

solutions: $(2, -1)$. $\qquad (-2, 3)$.

Sec 1.1

9. Given a system of the form

$$-m_1x_1 + x_2 = b_1$$
$$-m_2x_1 + x_2 = b_2$$

where m_1, m_2, b_1, and b_2 are constants:

(a) Show that the system will have a unique solution if $m_1 \neq m_2$.

(b) Show that if $m_1 = m_2$, then the system will be consistent only if $b_1 = b_2$.

(c) Give a geometric interpretation of parts (a) and (b).

(a). $-m_1X_1 + X_2 = b_1 \Rightarrow X_2 = m_1X_1 + b_1$ (Plug into 2nd eq)

$$\Rightarrow -m_2X_1 + m_1X_1 + b_1 = b_2$$

$$\Rightarrow (m_1 - m_2)\cdot X_1 = b_2 - b_1 \quad (*)$$

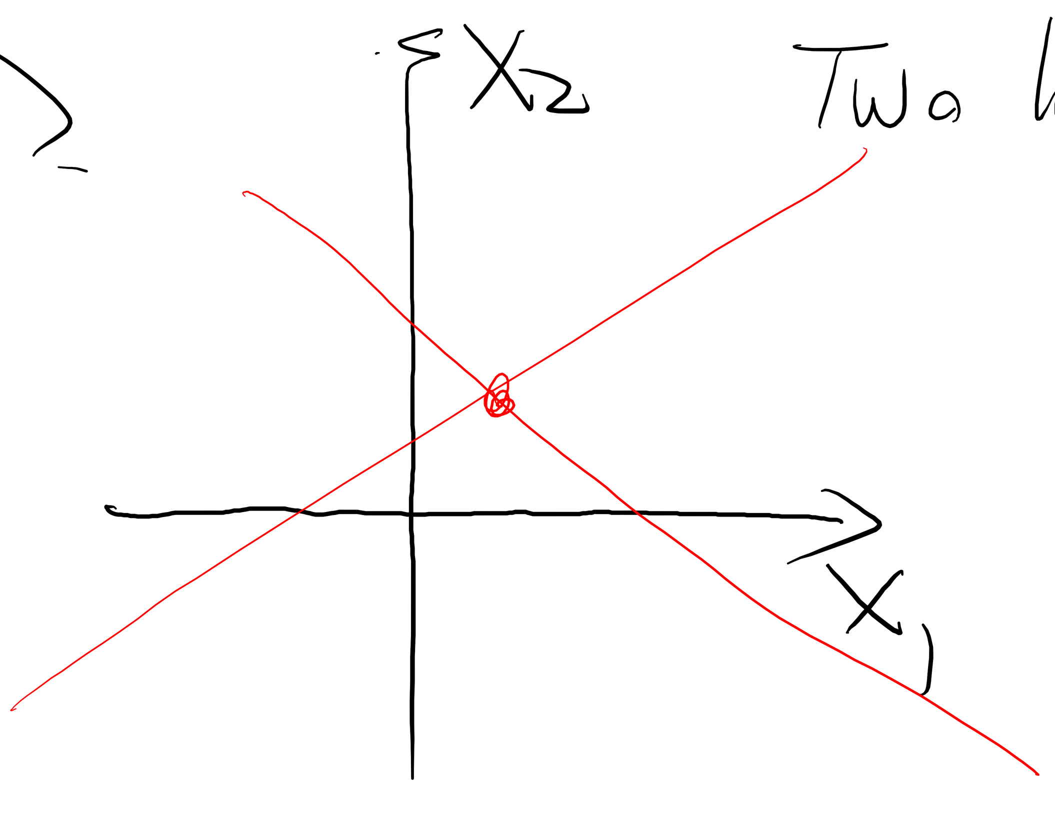
$$m_1 \neq m_2 \Rightarrow \boxed{X_1 = \frac{b_2 - b_1}{m_1 - m_2}} \Rightarrow \boxed{X_2 = m_1 \cdot \frac{b_2 - b_1}{m_1 - m_2} + b_1}$$

(b). If $m_1 = m_2$ and $b_1 \neq b_2$, then by $(*)$

$(m_1 - m_2)\cdot X_1 = 0 \neq b_2 - b_1$. no solution.

If $m_1 = m_2$ and $b_1 = b_2$, then the two equations are the same. The system has infinitely many solutions given by $X_2 = m_1 \cdot X_1 + b_1$

(c).

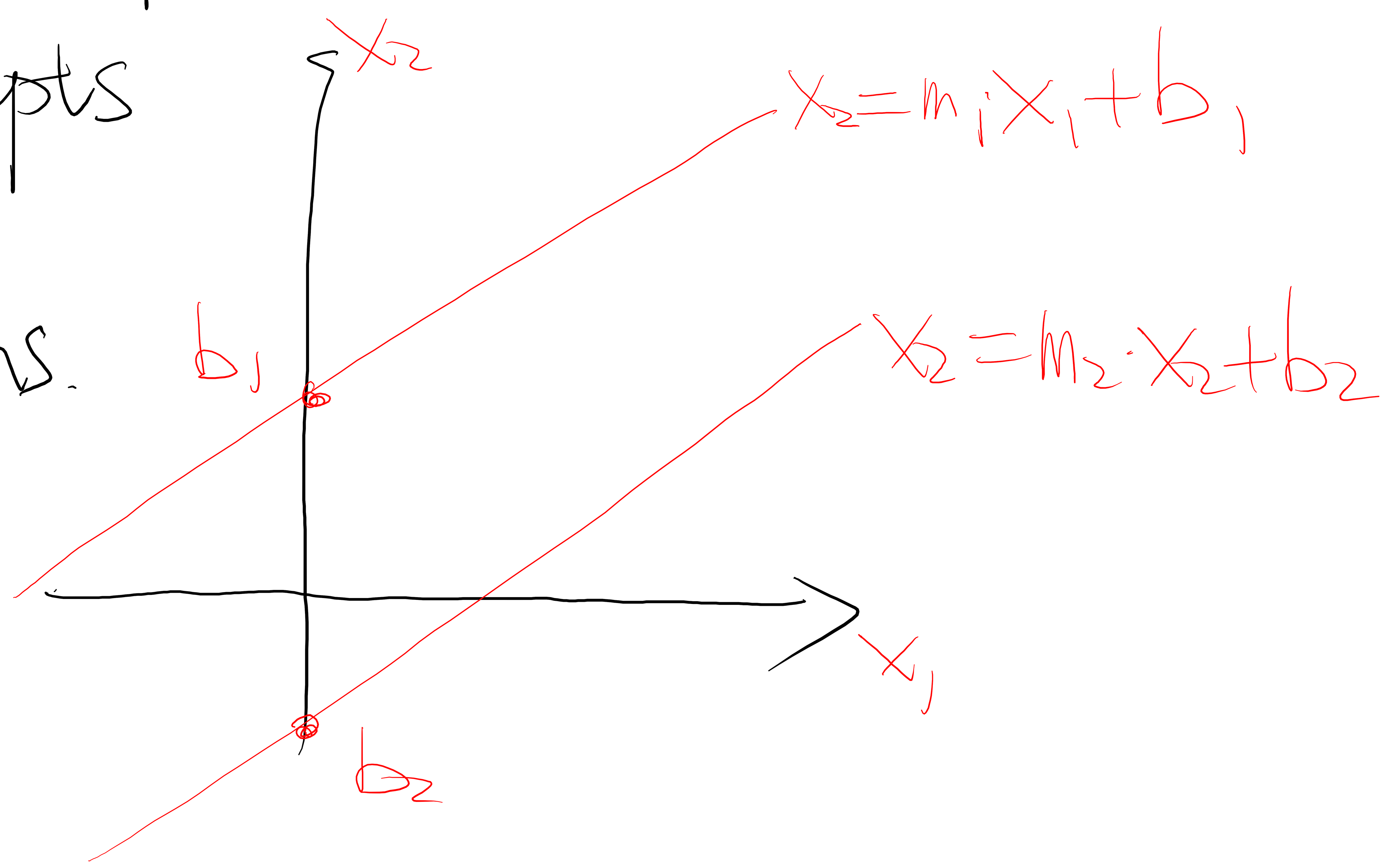


Two lines: $X_2 = m_1X_1 + b_1$, $X_2 = m_2X_1 + b_2$

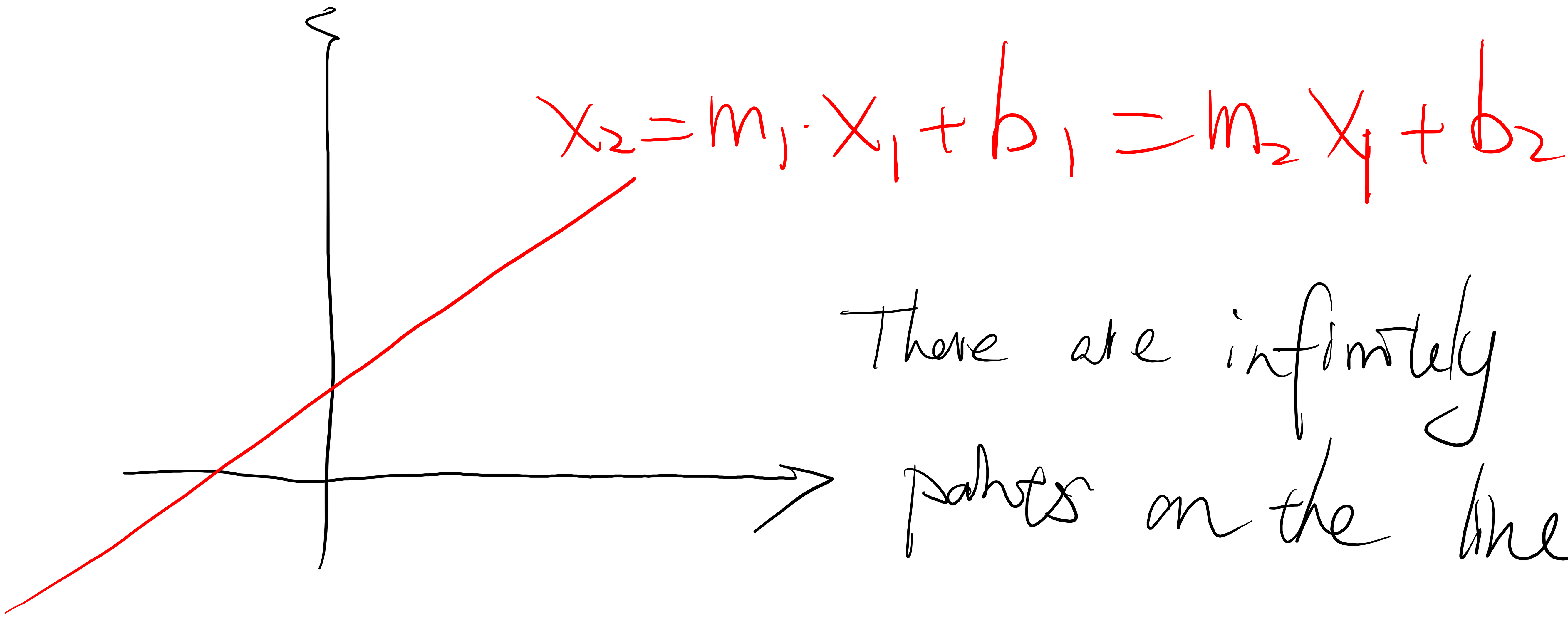
$m_1 \neq m_2$, then the two lines are not parallel to each other, and must intersect at one point.

If $m_1 = m_2$ and $b_1 \neq b_2$, then the two lines are parallel to each other with different x_2 intercepts

No intersections.



If $m_1 = m_2$ and $b_1 = b_2$, then the two lines are the same.



There are infinitely many points on the line.