HW3

2019年2月1日 16:51

Sec1.3, 1(d)(e)(h); 2(a)(b)(e); 4(a)(b)(c); 5 (c); 6 (c); 7(b); 8 (b)(c)(d)

1. If

9 pt

$$A = \begin{bmatrix} 3 & 1 & 4 \\ -2 & 0 & 1 \\ 1 & 2 & 2 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 1 & 0 & 2 \\ -3 & 1 & 1 \\ 2 & -4 & 1 \end{bmatrix}$$

compute

(a) $2A$ **(b)** $A + B$

(c) $2A - 3B$ **(d)** $(2A)^T - (3B)^T$

(e) AB **(f)** BA

(g) $A^T B^T$ **(h)** $(BA)^T$

(d). 3pt

$$(2A)^T - (3B)^T = \begin{bmatrix} 6 & 2 & 8 \\ -4 & 0 & 2 \\ 2 & 4 & 4 \end{bmatrix}^T - \begin{bmatrix} 3 & 0 & 6 \\ -9 & 3 & 3 \\ 6 & -12 & 3 \end{bmatrix}^T$$

1/3

$$= \begin{bmatrix} 6 & -4 & 2 \\ 2 & 0 & 4 \\ 8 & 2 & 4 \end{bmatrix} - \begin{bmatrix} 3 & -9 & 6 \\ 0 & 3 & -12 \\ 6 & 3 & 3 \end{bmatrix}$$

1/3

$$= \begin{bmatrix} 3 & 5 & -4 \\ 2 & -3 & 16 \\ 2 & -1 & 1 \end{bmatrix}$$

$1/3$

(e). 3pt.

$$A \cdot B = \begin{bmatrix} 3 & 1 & 4 \\ -2 & 0 & 1 \\ 1 & 2 & 2 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 2 \\ -3 & 1 & 1 \\ 2 & -4 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3\cdot1+1\cdot(-3)+4\cdot2= & (3\ 1\ 4)\begin{pmatrix}0\\1\\-4\end{pmatrix}=1-16= & (3\ 1\ 4)\begin{pmatrix}2\\1\\1\end{pmatrix}= \\ & & 6+1+4= \\ 8 & -15 & 11 \\ (-2\ 0\ 1)\begin{pmatrix}1\\-3\\2\end{pmatrix}= & (-2\ 0\ 1)\begin{pmatrix}0\\1\\-4\end{pmatrix}= & (-2\ 0\ 1)\begin{pmatrix}2\\1\\1\end{pmatrix}= \\ & & -4+1= \\ 0 & -4 & -3 \\ (1\ 2\ 2)\begin{pmatrix}1\\-3\\2\end{pmatrix}= & (1\ 2\ 2)\begin{pmatrix}0\\1\\-4\end{pmatrix}= & (1\ 2\ 2)\begin{pmatrix}2\\1\\1\end{pmatrix}= \\ 1-6+4= & 0+2-8= & 2+2+2= \\ -1 & -6 & 6 \end{bmatrix}$$

The red part is not needed for students' work.

1pt/3 if ≤ 3 numbers are correct (>0).

2pt/3 if ≤ 3 numbers are wrong (>0).

0pt if the dimension is wrong.

(h). 3pt

$$BA = \begin{bmatrix} 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 3 & 1 & 4 \end{bmatrix}$$

(h). 3pt

$$BA = \begin{bmatrix} 1 & 0 & 2 \\ -3 & 1 & 1 \\ 2 & -4 & 1 \end{bmatrix} \begin{bmatrix} 3 & 1 & 4 \\ -2 & 0 & 1 \\ 1 & 2 & 2 \end{bmatrix}.$$

$$= \begin{bmatrix} 5 & 5 & 8 \\ -10 & -1 & -9 \\ 15 & 4 & 6 \end{bmatrix}$$

1pt/3 if partially correct.

2pts/3 for correct BA

$$(BA)^T = \begin{bmatrix} 5 & -10 & 15 \\ 5 & -1 & 4 \\ 8 & -9 & 6 \end{bmatrix}$$

1pt/3 for correct transpose based on their BA.

5 pt **2.** For each of the pairs of matrices that follow, determine whether it is possible to multiply the first matrix times the second. If it is possible, perform the multiplication.

(a) $\begin{bmatrix} 3 & 5 & 1 \\ -2 & 0 & 2 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 1 & 3 \\ 4 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} 4 & -2 \\ 6 & -4 \\ 8 & -6 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 4 & 3 \\ 0 & 1 & 4 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} 3 & 2 \\ 1 & 1 \\ 4 & 5 \end{bmatrix}$

(d) $\begin{bmatrix} 4 & 6 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 3 & 1 & 5 \\ 4 & 1 & 6 \end{bmatrix}$

(e) $\begin{bmatrix} 4 & 6 & 1 \\ 2 & 1 & 1 \end{bmatrix} \begin{bmatrix} 3 & 1 & 5 \\ 4 & 1 & 6 \end{bmatrix}$

(f) $\begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix} \begin{bmatrix} 3 & 2 & 4 & 5 \end{bmatrix}$

3pt (a). Yes.

$$\begin{bmatrix} 3 & 5 & 1 \\ -2 & 0 & 2 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 1 & 3 \\ 4 & 1 \end{bmatrix} = \begin{bmatrix} 15 & 19 \\ 4 & 0 \end{bmatrix}$$

0 pt if size is wrong.

1/3 if ≤2 correct

2/3 if ≤2 wrong

$-1/3$ if ≤ 2 wrong

(b) (e). Can not

1pt each. No explanations needed.

4. Write each of the following systems of equations as a matrix equation:

(a)
$$\begin{aligned} 3x_1 + 2x_2 &= 1 \\ 2x_1 - 3x_2 &= 5 \end{aligned}$$

(b)
$$\begin{aligned} x_1 + x_2 \phantom{{}- x_3} &= 5 \\ 2x_1 + x_2 - x_3 &= 6 \\ 3x_1 - 2x_2 + 2x_3 &= 7 \end{aligned}$$

(c)
$$\begin{aligned} 2x_1 + x_2 + x_3 &= 4 \\ x_1 - x_2 + 2x_3 &= 2 \\ 3x_1 - 2x_2 - x_3 &= 0 \end{aligned}$$

(a). 2pt.

$$\underbrace{\begin{bmatrix} 3 & 2 \\ 2 & -3 \end{bmatrix}}_{1\text{pt}} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{pmatrix} 1 \\ 5 \end{pmatrix}$$

1pt

Or let

$$A = \begin{bmatrix} 3 & 2 \\ 2 & -3 \end{bmatrix}, \quad x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, \quad b = \begin{pmatrix} 1 \\ 5 \end{pmatrix}$$

$$Ax = b.$$

(b) 2pt.

$$\begin{bmatrix} 1 & 1 & 0 \\ 2 & 1 & -1 \\ 3 & -2 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{pmatrix} 5 \\ 6 \\ 7 \end{pmatrix}$$

1pt 1pt

1 pt 1 pt

or $A \quad x = b.$

(c)
2pt $\begin{bmatrix} 2 & 1 & 1 \\ 1 & -2 & 2 \\ 3 & -2 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \\ 0 \end{bmatrix}$

1pt 1pt

2pt **5.** If

$$A = \begin{bmatrix} 3 & 4 \\ 1 & 1 \\ 2 & 7 \end{bmatrix}$$

verify that

(a) $5A = 3A + 2A$ **(b)** $6A = 3(2A)$

(c) $(A^T)^T = A$

$A^T = \begin{bmatrix} 3 & 1 & 2 \\ 4 & 1 & 7 \end{bmatrix}$ 1pt/2

$(A^T)^T = \begin{bmatrix} 3 & 1 & 2 \\ 4 & 1 & 7 \end{bmatrix}^T$

$= \begin{bmatrix} 3 & 4 \\ 1 & 1 \\ 2 & 7 \end{bmatrix}$ 2

6. If

$$A = \begin{bmatrix} 4 & 1 & 6 \\ 2 & 3 & 5 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 1 & 3 & 0 \\ -2 & 2 & -4 \end{bmatrix}$$

verify that

(a) $A + B = B + A$

(b) $3(A + B) = 3A + 3B$

(c) $(A + B)^T = A^T + B^T$

(c). 2pt

$$A+B = \begin{bmatrix} 5 & 4 & 6 \\ 0 & 5 & 1 \end{bmatrix}$$

$$(A+B)^T = \begin{bmatrix} 5 & 0 \\ 4 & 5 \\ 6 & 1 \end{bmatrix}$$

1pt/2

$$A^T + B^T = \begin{bmatrix} 4 & 2 \\ 1 & 3 \\ 6 & 5 \end{bmatrix} + \begin{bmatrix} 1 & -2 \\ 3 & 2 \\ 0 & -4 \end{bmatrix}$$

2pt/2

$$= \begin{bmatrix} 5 & 0 \\ 4 & 5 \\ 6 & 1 \end{bmatrix} = (A+B)^T$$

7. If

$$A = \begin{bmatrix} 2 & 1 \\ 6 & 3 \\ -2 & 4 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 2 & 4 \\ 1 & 6 \end{bmatrix}$$

verify that

(a) $3(AB) = (3A)B = A(3B)$,

(b) $(AB)^T = B^T A^T$

(b) 4pt.

$$AB = \begin{bmatrix} 2 & 1 \\ 6 & 3 \\ -2 & 4 \end{bmatrix} \begin{bmatrix} 2 & 4 \\ 1 & 6 \end{bmatrix} = \begin{bmatrix} 5 & 14 \\ 15 & 42 \\ 0 & 16 \end{bmatrix}$$

2pt/4

$$B^T = \begin{bmatrix} 2 & 1 \\ 4 & 6 \end{bmatrix}, \quad A^T = \begin{bmatrix} 2 & 6 & -2 \\ 1 & 3 & 4 \end{bmatrix}$$

1pt/4

$$B^T \cdot A^T = \begin{bmatrix} 5 & 15 & 0 \\ 14 & 42 & 16 \end{bmatrix} = (AB)^T$$

1pt/4.

4 pt **8.** If

$$A = \begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix}, \quad B = \begin{bmatrix} -2 & 1 \\ 0 & 4 \end{bmatrix}, \quad C = \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$$

verify that

(a) $(A + B) + C = A + (B + C)$

(b) $(AB)C = A(BC)$

(c) $A(B + C) = AB + AC$

(d) $(A + B)C = AC + BC$

(b).

$$AB = \begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} -2 & 1 \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} -4 & 18 \\ -2 & 13 \end{bmatrix}$$

1pt/3

$$(AB)C = \begin{bmatrix} -4 & 18 \\ \ldots \end{bmatrix} \begin{bmatrix} 3 & 1 \\ \ldots \end{bmatrix} = \begin{bmatrix} 24 & 14 \\ \ldots \end{bmatrix}$$

$$(AB)\cdot C = \begin{bmatrix} -4 & 18 \\ -2 & 13 \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 24 & 14 \\ 20 & 11 \end{bmatrix}$$ 1pt/3

$$BC = \begin{bmatrix} -2 & 1 \\ 0 & 4 \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} -4 & -1 \\ 8 & 4 \end{bmatrix}$$ 1pt/3

$$A(BC) = \begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} -4 & -1 \\ 8 & 4 \end{bmatrix} = \begin{bmatrix} 24 & 14 \\ 20 & 11 \end{bmatrix}$$

$$= (AB)C$$

3pt (c). $$B + C = \begin{bmatrix} -2 & 1 \\ 0 & 4 \end{bmatrix} + \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & 5 \end{bmatrix}$$ 1pt/3

$$A(B+C) = \begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 10 & 24 \\ 7 & 17 \end{bmatrix}$$ 1pt/3

$$AB = \begin{bmatrix} -4 & 18 \\ -2 & 13 \end{bmatrix}$$ (from (a)).

$$AC = \begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 14 & 6 \\ 9 & 4 \end{bmatrix}$$ 1pt/3

$$AB + AC = \begin{bmatrix} 10 & 24 \\ 7 & 17 \end{bmatrix} = A(B+C)$$

(d) $A(B - \ldots = \begin{bmatrix} 2 & 4 \end{bmatrix} \begin{bmatrix} -2 & 1 \end{bmatrix} \begin{bmatrix} 0 & 5 \end{bmatrix}$

(d)

3pt

$$A+B=\begin{bmatrix}2 & 4\\ 1 & 3\end{bmatrix}+\begin{bmatrix}-2 & 1\\ 0 & 4\end{bmatrix}=\begin{bmatrix}0 & 5\\ 1 & 7\end{bmatrix}$$

$$(A+B)C=\begin{bmatrix}0 & 5\\ 1 & 7\end{bmatrix}\cdot\begin{bmatrix}3 & 1\\ 2 & 1\end{bmatrix}=\begin{bmatrix}10 & 5\\ 17 & 8\end{bmatrix}$$ 1pt/3

$$A\cdot C=\begin{bmatrix}2 & 4\\ 1 & 3\end{bmatrix}\cdot\begin{bmatrix}3 & 1\\ 2 & 1\end{bmatrix}=\begin{bmatrix}14 & 6\\ 9 & 4\end{bmatrix}$$ 1pt/3

$$B\cdot C=\begin{bmatrix}-4 & -1\\ 8 & 4\end{bmatrix}$$ (from (a))

$$AC+BC=\begin{bmatrix}14 & 6\\ 9 & 4\end{bmatrix}+\begin{bmatrix}-4 & -1\\ 8 & 4\end{bmatrix}$$

$$=\begin{bmatrix}10 & 5\\ 17 & 8\end{bmatrix}=(A+B)\cdot C$$ 1pt/3.

Sec 1.4, 7, 9, 11(a)(c)(e)

来自 <https://users.math.msu.edu/users/zhangshiwen/s19/homework.html>

7. Let

$$A=\begin{bmatrix}\frac{1}{2} & -\frac{1}{2}\\ -\frac{1}{2} & \frac{1}{2}\end{bmatrix}$$

Compute A^2 and A^3 . What will A^n turn out to be?

7. $A^2 = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{bmatrix}$

3pt.

$$= \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{bmatrix}$$ 1pt/3

$$A^3 = A^2 \cdot A = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{bmatrix}$$ 1pt/3

$A^n = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{bmatrix}$ for any n. 1pt/3

9. Let

4pt

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Show that $A^n = O$ for $n \geq 4$.

$$A^2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0&0&0&1\\ 0&0&0&0 \end{bmatrix}\begin{bmatrix} 0&0&0&1\\ 0&0&0&0 \end{bmatrix}$$

$$= \begin{bmatrix} 0&0&1&0\\ 0&0&0&1\\ 0&0&0&0\\ 0&0&0&0 \end{bmatrix}$$

1 pt/4

$$A^3 = A^2 \cdot A = \begin{bmatrix} 0&0&1&0\\ 0&0&0&1\\ 0&0&0&0\\ 0&0&0&0 \end{bmatrix} \cdot \begin{bmatrix} 0&1&0&0\\ 0&0&1&0\\ 0&0&0&1\\ 0&0&0&0 \end{bmatrix}$$

$$= \begin{bmatrix} 0&0&0&1\\ 0&0&0&0\\ 0&0&0&0\\ 0&0&0&0 \end{bmatrix}$$

1pt/4

$$A^4 = A^3 \cdot A$$

$$= \begin{bmatrix} 0&\cdots&1\\ & 0 & \\ 0&\cdots&0 \end{bmatrix}\begin{bmatrix} 0&1&0\\ &0&1\\ 0&0&0 \end{bmatrix} = \begin{bmatrix} 0&\cdots&0\\ \vdots&0&\vdots\\ 0&\cdots&0 \end{bmatrix} = 0$$

1pt/4

Therefore, for all $n \geq 4$.

$$A^n = 0$$

1pt/4.

3 pt **11.** Let C be a nonsymmetric $n \times n$ matrix. For each of the following, determine whether the given matrix must necessarily be symmetric or could possibly be nonsymmetric:

(a) $A = C + C^T$ **(b)** $B = C - C^T$

(c) $D = C^T C$ **(d)** $E = C^T C - CC^T$

(e) $F = (I + C)(I + C^T)$

(c) $D = C^TC$ (d) $E = C^TC - CC^T$

(e) $F = (I + C)(I + C^T)$

1pt/2

1 pt (a). $A = C + C^T$ is symmetric (since

$$A^T = (C + C^T)^T = C^T + (C^T)^T = C^T + C = A.$$

or write $C = (c_{ij})$

$$A = (a_{ij}) = (c_{ij}) + (c_{ji}) = (c_{ij} + c_{ji})$$

$$a_{ij} = c_{ij} + c_{ji} = a_{ji}$$)

1pt (c). $D = C^T \cdot C$ is symmetric 1pt/1

since $D^T = (C^T C)^T = C^T \cdot (C^T)^T$

$$= C^T \cdot C = D.$$

1pt (e). $F = (I + C)(I + C^T)$

is symmetric 1pt/1.

since

$$F = (I+C)(I+C^T)$$
$$= I \cdot I + I \cdot C^T + C \cdot I + C \cdot C^T$$
$$= I + C^T + C + C \cdot C^T$$
$$F^T = I^T + (C^T)^T + C^T + (C \cdot C^T)^T$$
$$= I + C + C^T + (C^T)^T \cdot C^T$$
$$= I + C + C^T + C \cdot C^T = F.$$

No explanation need for (a)(c)(e).

due to change of lectures schedule caused by the storm.