Hw2

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Sec1.2, 3(a)(b)(c)(d), 4(a)(b)(c)(d), 5 (e)(f)

3. The augmented matrices that follow are in reduced row echelon form. In each case, find the solution set to the corresponding linear system.

(a) $\left[\begin{array}{ccc|c} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 3 \end{array}\right]$ **(b)** $\left[\begin{array}{ccc|c} 1 & 4 & 0 & 2 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{array}\right]$

(c) $\left[\begin{array}{ccc|c} 1 & -3 & 0 & 2 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 \end{array}\right]$

(d) $\left[\begin{array}{cccc|c} 1 & 2 & 0 & 1 & 5 \\ 0 & 0 & 1 & 3 & 4 \end{array}\right]$

(a). 2pt for correct answer.

$$\begin{cases} x_1 = -2 \\ x_2 = 5 \\ x_3 = 3 \end{cases} \quad \text{or} \quad (-2, 5, 3).$$

2pt for correct answer

(b). $\begin{cases} x_1 + 4x_2 = 2 \\ x_3 = 3 \\ 0 = 1. \end{cases}$ no solution (inconsistent)

(c). 3pt. $\begin{cases} x_1 - 3x_2 = 2 \\ x_3 = -2 \end{cases} \Rightarrow \begin{cases} x_1 = 3x_2 + 2 \\ x_3 = -2. \end{cases}$ 1pt/3

Solution: $(3\alpha+2, \alpha, -2)$

or $(3x_2+2, x_2, -2)$. 2pt/3

(d). 3pt
$$\begin{cases} x_1 + 2x_2 + x_4 = 5 \\ x_3 + 3x_4 = 4. \end{cases}$$

1pt/3

$$\Rightarrow \begin{cases} x_1 = -2x_2 - x_4 + 5 \\ x_3 = -3x_4 + 4. \end{cases}$$

Solution: $(-2\alpha-\beta+5, \alpha, -3\beta+4, \beta)$ 2pt/3

or $(-2x_2-x_4+5, x_2, -3x_4+4, x_4)$.

8pt total

4. For each of the systems in Exercise 3, make a list of the lead variables and a second list of the free variables. (a) (b) (c) (d).

(a). Lead variables: x_1, x_2, x_3

No free variables.

(b). Inconsistent. No lead and no free variables.

(also correct if they say x_1, x_3 lead and x_2 free).

(c). Lead v.: x_1, x_3

Free v.: x_2.

Free v.: x_2.

(d). Lead v: x_1, x_3.

Free v: x_2, x_4

2pt each. 1pt if partially correct.

5. For each of the systems of equations that follow, use Gaussian elimination to obtain an equivalent system whose coefficient matrix is in row echelon form. Indicate whether the system is consistent. If the system is consistent and involves no free variables, use back substitution to find the unique solution. If the system is consistent and there are free variables, transform it to reduced row echelon form and find all solutions. (e) (f).

(e)
$$\begin{aligned} 2x_1 + 3x_2 + x_3 &= 1 \\ x_1 + x_2 + x_3 &= 3 \\ 3x_1 + 4x_2 + 2x_3 &= 4 \end{aligned}$$

(e). 5pt
$$\left[\begin{array}{ccc|c} 2 & 3 & 1 & 1 \\ 1 & 1 & 1 & 3 \\ 3 & 4 & 2 & 4 \end{array}\right]$$
correct matrix +1/5

$$\xrightarrow{①\leftrightarrow②} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 2 & 3 & 1 & 1 \\ 3 & 4 & 2 & 4 \end{array}\right]$$

$$\xrightarrow[\text{③}-3\cdot\text{①}]{\text{②}-2\cdot\text{①}} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & 1 & -1 & -5 \\ 0 & 1 & -1 & -5 \end{array}\right]$$

$$\xrightarrow{\text{③}-\text{②}} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & 1 & -1 & -5 \\ 0 & 0 & 0 & 0 \end{array}\right]$$ row echelon form. with free variable.

correct R.E.F +2/5

$$\xrightarrow{\text{①}-\text{②}} \left[\begin{array}{ccc|c} 1 & 0 & 2 & 8 \\ 0 & 1 & -1 & -5 \\ 0 & 0 & 0 & 0 \end{array}\right]$$ reduced R.E.F.

correct R.E.F. +1/5

$$\begin{cases} x_1 + 2x_3 = 8 \\ x_2 - x_3 = -5 \end{cases} \Rightarrow \begin{cases} x_1 = -2x_3 + 8 \\ x_2 = x_3 - 5 \end{cases}$$

solution : $(-2x_3+8,\ x_3-5,\ x_3)$

or $(-2\alpha+8,\ \alpha-5,\ \alpha)$.

correct solution +1/5

(f).

5 pt

(f) $$\begin{aligned} x_1 - x_2 + 2x_3 &= 4 \\ 2x_1 + 3x_2 - x_3 &= 1 \\ 7x_1 + 3x_2 + 4x_3 &= 7 \end{aligned}$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 2 & 4 \\ 2 & 3 & -1 & 1 \\ 7 & 3 & 4 & 7 \end{array}\right]$$

+1/5

$$\xrightarrow[③-7\cdot①]{②-2①} \left[\begin{array}{ccc|c} 1 & -1 & 2 & 4 \\ 0 & 5 & -5 & -7 \\ 0 & 10 & -10 & -21 \end{array}\right]$$

$$\xrightarrow{③-2②} \left[\begin{array}{ccc|c} 1 & -1 & 2 & 4 \\ 0 & 5 & -5 & -7 \\ 0 & 0 & 0 & -7 \end{array}\right]$$

$$\xrightarrow{②/5} \left[\begin{array}{ccc|c} 1 & -1 & 2 & 4 \\ 0 & 1 & -1 & -\frac{7}{5} \\ 0 & 0 & 0 & -7 \end{array}\right]$$

Correct R.E.F. +2/5.

Row echelon form. Inconsistent since $0=-7$.

correct conclusion +2/5.

(It's ok if they also reduce the last row to $0 \quad 0 \quad 0 \quad 1$).

Sec1.3, 1(a)(b)(c); 5(a)(b); 6(a)(b); 8(a)

1. If

$$A = \begin{bmatrix} 3 & 1 & 4 \\ -2 & 0 & 1 \\ 1 & 2 & 2 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 1 & 0 & 2 \\ -3 & 1 & 1 \\ 2 & -4 & 1 \end{bmatrix}$$

compute

(a) $2A$ **(b)** $A + B$

(c) $2A - 3B$ ~~**(d)** $(2A)^T - (3B)^T$~~

1 pt each, 3pt total.

(a). $2A = \begin{bmatrix} 6 & 2 & 8 \\ -4 & 0 & 2 \\ 2 & 4 & 4 \end{bmatrix}$

(b) $A+B = \begin{bmatrix} 4 & 1 & 6 \\ -5 & 1 & 2 \\ 3 & -2 & 3 \end{bmatrix}$

(c). $2A-3B = \begin{bmatrix} 6 & 2 & 8 \\ -4 & 0 & 2 \\ 2 & 4 & 4 \end{bmatrix} - \begin{bmatrix} 3 & 0 & 6 \\ -9 & 3 & 3 \\ 6 & -12 & 3 \end{bmatrix}$

$= \begin{bmatrix} 3 & 2 & 2 \\ 5 & -3 & -1 \\ -4 & 16 & 1 \end{bmatrix}$

5. If

$$A = \begin{bmatrix} 3 & 4 \\ 1 & 1 \\ 2 & 7 \end{bmatrix}$$

verify that

(a) $5A = 3A + 2A$ **(b)** $6A = 3(2A)$

3pt each. They need to compute both sides

3pt each. They need to compute both sides with details.

(a). $L.H.S. = 5A = \begin{bmatrix} 15 & 20 \\ 5 & 5 \\ 10 & 35 \end{bmatrix}$. +1/3

$R.H.S = 2A + 3A.$

$$= \begin{bmatrix} 6 & 8 \\ 2 & 2 \\ 4 & 14 \end{bmatrix} + \begin{bmatrix} 9 & 12 \\ 3 & 3 \\ 6 & 21 \end{bmatrix} = \begin{bmatrix} 15 & 20 \\ 5 & 5 \\ 10 & 35 \end{bmatrix}$$

+1/3 +1/3.

(b). $6A = \begin{bmatrix} 18 & 24 \\ 6 & 6 \\ 12 & 42 \end{bmatrix}$ +1/3

$$3(2A) = 3 \cdot \begin{bmatrix} 6 & 8 \\ 2 & 2 \\ 4 & 14 \end{bmatrix} = \begin{bmatrix} 18 & 24 \\ 12 & 12 \\ 12 & 42 \end{bmatrix}$$

+1/3 +1/3.

6. If

$$A = \begin{bmatrix} 4 & 1 & 6 \\ 2 & 3 & 5 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 3 & 0 \\ -2 & 2 & -4 \end{bmatrix}$$

verify that

(a) $A + B = B + A$

(b) $3(A + B) = 3A + 3B$

(a). $A+B=\begin{bmatrix}4&1&6\\2&3&5\end{bmatrix}+\begin{bmatrix}1&3&0\\-2&2&-4\end{bmatrix}$

1pt

$$=\begin{bmatrix}5&4&6\\0&5&1\end{bmatrix}$$ +1.

$$=B+A$$

(b). $3(A+B)=3\begin{bmatrix}5&4&6\\0&5&1\end{bmatrix}$

3pt

$$=\begin{bmatrix}15&12&18\\0&15&3\end{bmatrix}$$ +1/3

$$3A+3B=\begin{bmatrix}12&3&18\\6&9&15\end{bmatrix}+\begin{bmatrix}3&9&0\\-6&6&-12\end{bmatrix}$$

+1/3 +1/3

$$=\begin{bmatrix}15&12&18\\0&15&3\end{bmatrix}$$

8. If

3pt. $A=\begin{bmatrix}2&4\\1&3\end{bmatrix}$, $B=\begin{bmatrix}-2&1\\0&4\end{bmatrix}$, $C=\begin{bmatrix}3&1\\2&1\end{bmatrix}$

verify that

(a) $(A+B)+C=A+(B+C)$

$$A+B=\begin{bmatrix}2&4\\1&3\end{bmatrix}+\begin{bmatrix}-2&1\\0&4\end{bmatrix}$$

$$=\begin{bmatrix}0&5\\1&7\end{bmatrix}$$ +1/3

$$(A+B)+C=\begin{bmatrix}0&5\\1&7\end{bmatrix}+\begin{bmatrix}3&1\\2&1\end{bmatrix}$$

$$=\begin{bmatrix}3&6\\3&8\end{bmatrix}$$ +1/3

$$B+C=\begin{bmatrix}-2&1\\0&4\end{bmatrix}+\begin{bmatrix}3&1\\2&1\end{bmatrix}=\begin{bmatrix}1&2\\2&5\end{bmatrix}$$ +1/3.

$$A+(B+C)=\begin{bmatrix}2&4\\1&3\end{bmatrix}+\begin{bmatrix}1&2\\2&5\end{bmatrix}=\begin{bmatrix}3&6\\3&8\end{bmatrix}$$

$$=(A+B)+C$$