

# HW1

2019年1月20日 22:44

## SECTION 1.1 EXERCISES

1. Use back substitution to solve each of the following systems of equations:

$$\text{(a)} \quad \begin{aligned} x_1 - 3x_2 &= 2 \\ 2x_2 &= 6 \end{aligned} \quad \text{(b)} \quad \begin{aligned} x_1 + x_2 + x_3 &= 8 \\ 2x_2 + x_3 &= 5 \end{aligned}$$

$$3x_3 = 9$$

$$\text{(c)} \quad \begin{aligned} x_1 + 2x_2 + 2x_3 + x_4 &= 5 \\ 3x_2 + x_3 - 2x_4 &= 1 \\ -x_3 + 2x_4 &= -1 \\ 4x_4 &= 4 \end{aligned}$$

$$\text{(a).} \quad x_2 = 3 \Rightarrow x_1 = 11$$

Solution:  $(11, 3)$

$$\text{(c).} \quad x_4 = 1 \Rightarrow -x_3 + 2 = -1 \Rightarrow x_3 = 3$$

$$\Rightarrow 3x_2 + 3 - 2 = 1 \Rightarrow x_2 = 0$$

$$\Rightarrow x_1 + 0 + 6 + 1 = 5 \Rightarrow x_1 = -2$$

Solution:  $(-2, 0, 3, 1)$

2. Write out the coefficient matrix for each of the systems in Exercise 1. (a), (c).

$$\text{(a).} \quad \begin{bmatrix} 1 & -3 \\ 0 & 2 \end{bmatrix}$$

$$\text{(c)} \quad \begin{bmatrix} 1 & 2 & 2 & 1 \\ 0 & 3 & 1 & -2 \\ 0 & 0 & -1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & -1 & 2 \\ 0 & 0 & 0 & 4 \end{bmatrix}$$

3. In each of the following systems, interpret each equation as a line in the plane. For each system, graph the lines and determine geometrically the number of solutions.

(a)  $x_1 + x_2 = 4$

$$x_1 - x_2 = 2$$

(c)  $2x_1 - x_2 = 3$

$$-4x_1 + 2x_2 = -6$$

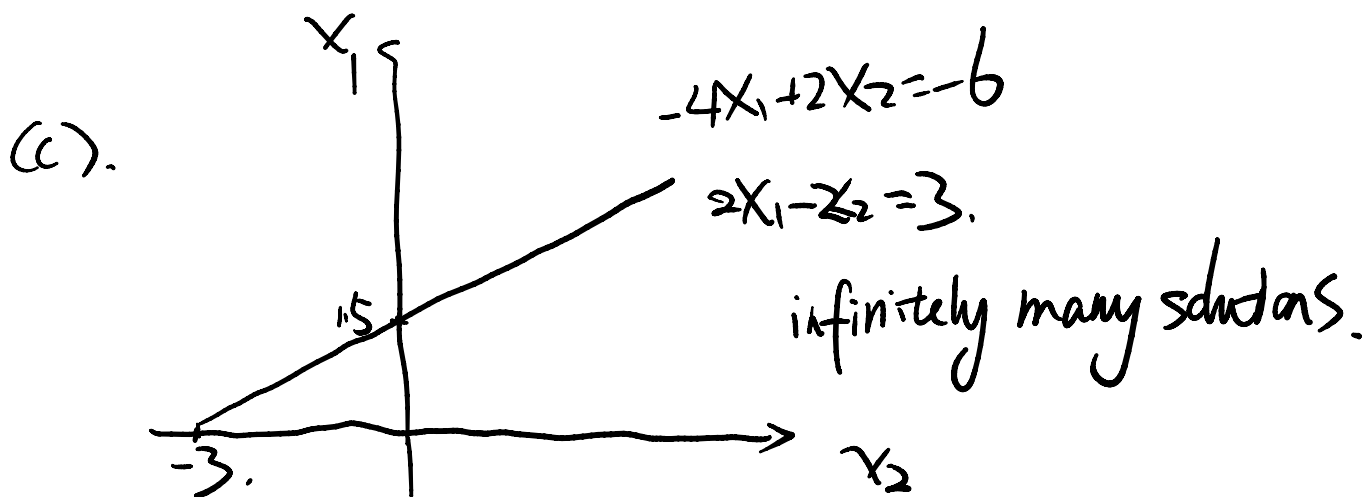
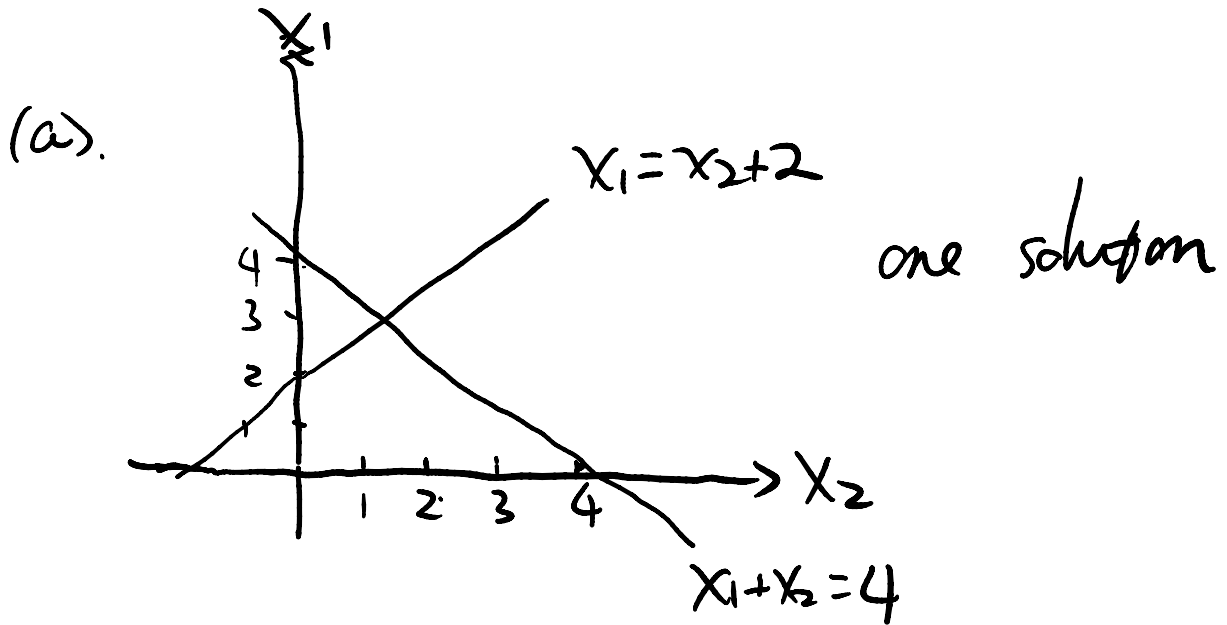
(b)  $x_1 + 2x_2 = 4$

$$-2x_1 - 4x_2 = 4$$

(d)  $x_1 + x_2 = 1$

$$x_1 - x_2 = 1$$

$$-x_1 + 3x_2 = 3$$



4. Write an augmented matrix for each of the systems in Exercise 3. (a) (c)

$$(a). \left[ \begin{array}{cc|c} 1 & 1 & 4 \\ 1 & -1 & 2 \end{array} \right]$$

$$(c). \left[ \begin{array}{cc|c} 2 & -1 & 3 \\ -4 & 2 & -6 \end{array} \right]$$

5. Write out the system of equations that corresponds to each of the following augmented matrices:

$$(c) \left[ \begin{array}{ccc|c} 2 & 1 & 4 & -1 \\ 4 & -2 & 3 & 4 \\ 5 & 2 & 6 & -1 \end{array} \right]$$

$$\left\{ \begin{array}{l} 2x_1 + x_2 + 4x_3 = -1 \\ 4x_1 - 2x_2 + 3x_3 = 4 \\ 5x_1 + 2x_2 + 6x_3 = -1 \end{array} \right.$$

6. Solve each of the following systems.

$$(e) \quad 2x_1 + x_2 + 3x_3 = 1$$

$$4x_1 + 3x_2 + 5x_3 = 1$$

$$6x_1 + 5x_2 + 5x_3 = -3$$

$$\left[ \begin{array}{ccc|c} 2 & 1 & 3 & 1 \\ 4 & 3 & 5 & 1 \\ 6 & 5 & 5 & -3 \end{array} \right] \rightarrow \left[ \begin{array}{ccc|c} 2 & 1 & 3 & 1 \\ 0 & 1 & -1 & -1 \\ 0 & 2 & -4 & -6 \end{array} \right]$$

$$\rightarrow \left[ \begin{array}{ccc|c} 2 & 1 & 3 & 1 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & -1 & -2 \end{array} \right] \quad \begin{array}{l} 2x_1 + x_2 + 3x_3 = 1 \\ x_2 - x_3 = -1 \\ -x_3 = -2. \end{array}$$

$$\Rightarrow x_3 = 2, \quad x_2 = 1, \quad 2x_1 + 1 + 6 = 1 \Rightarrow x_1 = -3$$

solution:  $(-3, 1, 2)$ .