List of key points for Final Exam, MTH 309, Last updated April 23, 2019

Chap. 1,2 $\sim 20\%$; Chap. 3,4,6 $\sim 30\%$; Chap. 5 $\sim 50\%$;

- Sec 1.1
  - (Solving) Linear system; Consistent and inconsistent system
  - Coefficient matrix and augmented matrix; Elementary row operations
- Sec 1.2
  - **(Reduced) Row echelon form**
  - Free and lead variables
- Sec 1.3
  - Matrix equations
  - Matrix arithmetic (addition/multiplication/transpose); Symmetric matrix
- Sec 1.4
  - Matrix algebra (algebraic rules for matrices); Matrix power; Indentity matrix;
  - Matrix inverse (singular and nonsingular matrix);
- Sec 1.5
  - Three types of elementary matrices; their relations to elementary row operations;
  - **Use elementary row operation to find the inverse of a 3×3 matrix**
- Sec 2.1
  - Definition of the determinant/cofactor
  - Determinant of transpose, triangular, and diagonal matrices
- Sec 2.2
  - Determinant of elementary matrices, and matrix product
  - Determinant of singular and nonsingular matrix
- Sec 3.1
  - Euclidean vector spaces $\mathbb{R}^n$
  - Polynomial vector spaces P_n
- Sec 3.2
  - Subspace of $\mathbb{R}^n$ and P_n
  - **(Finding) Null space of an $m \times n$ matrix**
  - Linear combination and the span of vectors $v_1, \cdots, v_n$ (in $\mathbb{R}^n$ and P_n)
  - Using determinant to check whether n vectors $v_1, \cdots, v_n$ in $\mathbb{R}^n$ span $\mathbb{R}^n$ or not
- Sec 3.3
  - Linear dependence/independence in $\mathbb{R}^n$ and P_n
  - Using determinant to check whether n vectors $v_1, \cdots, v_n$ in $\mathbb{R}^n$ are linearly independent or not
- Sec 3.4
  - Basis and dimension of $\mathbb{R}^n$ and P_n
  - Basis and dimension of subspaces of $\mathbb{R}^n$ and P_n

- Sec 3.5
  - Transition matrix from one basis to another in $\mathbb{R}^n$
- Sec 3.6
  - Definition of the rank and the nullity of an $m \times n$ matrix
  - The Rank-Nullity Theorem
- Sec 6.1
  - Definition of eigenvalues and eigenvectors
  - **Finding eigenvalues, eigenvectors and eigenspaces of 2×2 and 3×3 matrices**
  - The product and sum of eigenvalues and their relation to determinants and traces
- Sec 6.3
  - **Diagonalization using eigenvalues and eigenvectors for 2×2 and 3×3 matrices**
  - Computing the power of a matrix using diagonalization
- Sec 4.1
  - Linear transformation from $\mathbb{R}^n$ to $\mathbb{R}^m$
  - Kernel and range of a linear transformation from $\mathbb{R}^n$ to $\mathbb{R}^m$
- Sec 4.2
  - **Matrix representation of a linear transformation from $\mathbb{R}^n$ to $\mathbb{R}^m$**
- Sec 5.1
  - Scalar (inner) product and length (norm)in $\mathbb{R}^n$;
  - Distance and angles between vectors in $\mathbb{R}^n$;
  - Orthogonal vectors and scalar/vector projection
- Sec 5.2
  - Orthogonal subspaces
  - **Orthogonal compliment of a subspace spanned by several vectors**
- Sec 5.3
  - **Least squares solutions to a inconsistent system**
- Sec 5.4
  - Inner product in $\mathbb{R}^n$ and the Pythagorean law
  - Different norms in $\mathbb{R}^n$
- Sec 5.5
  - Orthogonal set; orthonormal set; orthonormal basis; orthogonal matrix
- Sec 5.6
  - **Gram-Schmidt process for finding orthonormal basis**