## List of key points for Mid 2, MTH 309

- Sec 3.1
  - Euclidean vector spaces  $\mathbb{R}^n$
  - Polynomial vector spaces  $P_n$
  - Verifying whether a given set is a vector space or not by the definition (C1,C2 and A1-A8 will be provided).
- Sec 3.2
  - Definition of vector subspace;
  - Subspace of  $\mathbb{R}^n$  and  $P_n$
  - Null space of an  $m \times n$  matrix
  - Linear combination and the span of vectors  $v_1, \dots, v_n$  (in  $\mathbb{R}^n$  and  $P_n$ )
  - Spanning set of  $\mathbb{R}^n$ ,  $P_n$  and their subspaces
  - Using determinant to check whether n vectors  $v_1, \dots, v_n$  in  $\mathbb{R}^n$  span  $\mathbb{R}^n$  or not
- Sec 3.3
  - Linear dependence/independence in  $\mathbb{R}^n$  and  $P_n$
  - Using determinant to check whether n vectors  $v_1, \cdots, v_n$  in  $\mathbb{R}^n$  are linearly independent or not
- $\bullet~$  Sec 3.4
  - Basis and dimension of  $\mathbb{R}^n$  and  $P_n$
  - Basis and dimension of subspaces of  $\mathbb{R}^n$  and  $P_n$
- $\bullet~$  Sec 3.5
  - Transition matrix from one basis to another in  $\mathbb{R}^n$
  - Changing coordinates using transition matrix in  $\mathbb{R}^n$
- Sec 3.6
  - Definition of the rank and the nullity of an  $m \times n$  matrix
  - The Rank-Nullity Theorem
- Sec 6.1
  - Definition of eigenvalues and eigenvectors
  - Finding eigenvalues, eigenvectors and eigenspaces of  $2 \times 2$  and  $3 \times 3$  matrices
  - The product and sum of eigenvalues and their relation to determinants and traces
- Sec 6.3
  - Diagonalization using eigenvalues and eigenvectors for  $2 \times 2$  and  $3 \times 3$  matrices
  - Computing the power of a matrix using diagonalization
- Sec 4.1
  - Linear transformation from  $\mathbb{R}^n$  to  $\mathbb{R}^m$
  - Linear transformation from  $P_n$  to  $P_m$
  - Kernel and range of a linear transformation from  $\mathbb{R}^n$  to  $\mathbb{R}^m$
- Sec 4.2
  - Matrix representation of a linear transformation from  $\mathbb{R}^n$  to  $\mathbb{R}^m$