Name: ______________________ **ID:** ______________

Clear your desk of everything except pens, pencils and erasers. Show all work clearly and in order. No notes, phones and calculators. You have 10 minutes to finish the test for 10 points. **ONLY THE PROBLEMS ON THIS PAGE COUNT. The problems on the back are not required, but will be graded if you finish them.**

Algebraic Formulas:

$$a^2 - b^2 = (a+b)(a-b); \qquad a^3 - b^3 = (a-b)(a^2+ab+b^2)$$

1. (a) (2 points) Evaluate the limit

$$\lim_{x\to\infty} \frac{9x-2}{5-2x}$$

$= \lim_{x\to\infty} \frac{9x}{-2x} = \lim_{x\to\infty} \frac{9}{-2} = \boxed{-\frac{9}{2}}$

Remark: Actually, $\frac{9x-2}{5-2x} = \frac{x(9-\frac{2}{x})}{x(\frac{5}{x}-2)} = \frac{9-\frac{2}{x}}{\frac{5}{x}-2}$.

$\lim_{x\to\infty} \frac{2}{x} = 0$, $\lim_{x\to\infty} \frac{5}{x} = 0$

$\Rightarrow \lim_{x\to\infty} \frac{9x-2}{5-2x} = \frac{9-0}{0-2} = -\frac{9}{2}$

(b) (2 points) Which of the following is the equation of a horizontal asymptote for the curve

$$y = \frac{9x-2}{5-2x}$$

A. $y = \frac{9}{2}$; **B.** $y = -\frac{9}{2}$; **C.** $y = 0$; **D.** $y = \frac{5}{2}$; **E.** $y = -\frac{5}{2}$

According to (a)

$\lim_{x\to\infty} \frac{9x-2}{5-2x} = -\frac{9}{2}$

$\Rightarrow y = -\frac{9}{2}$ is a H.A.

(c) (2 points) Which of the following is the equation of a vertical asymptote for the curve

$$y = \frac{9x-2}{5-2x}$$

A. $y = \frac{9}{2}$; **B.** $y = -\frac{9}{2}$; **C.** $y = 0$; **D.** $y = \frac{5}{2}$; **E.** $y = -\frac{5}{2}$

Set $5-2x=0$

$\Rightarrow x = \frac{5}{2}$ is a V.A.

2. (4 points) Write down the slant asymptote of the curve

$$y = \frac{x^2}{x-1}$$

$$\begin{array}{r} x + 1 \\ x-1 \overline{) x^2 + 0 + 0} \\ \underline{x^2 - x} \\ x + 0 \\ \underline{x - 1} \\ 1 \end{array}$$

$\Rightarrow x^2 = (x+1)(x-1) + 1$

$\Rightarrow \frac{x^2}{x-1} = \boxed{x+1} + \frac{1}{x-1}$

$y = x+1$ is a slant asymptote.

Remark: The following formula also works:

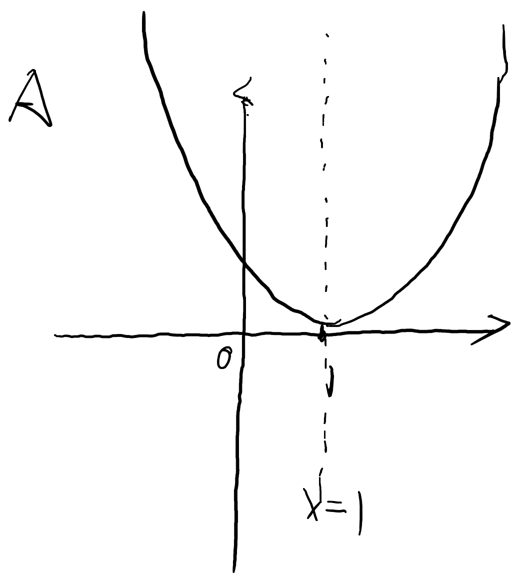
$y = \frac{x^2}{x-1} = \frac{x^2-1+1}{x-1}$

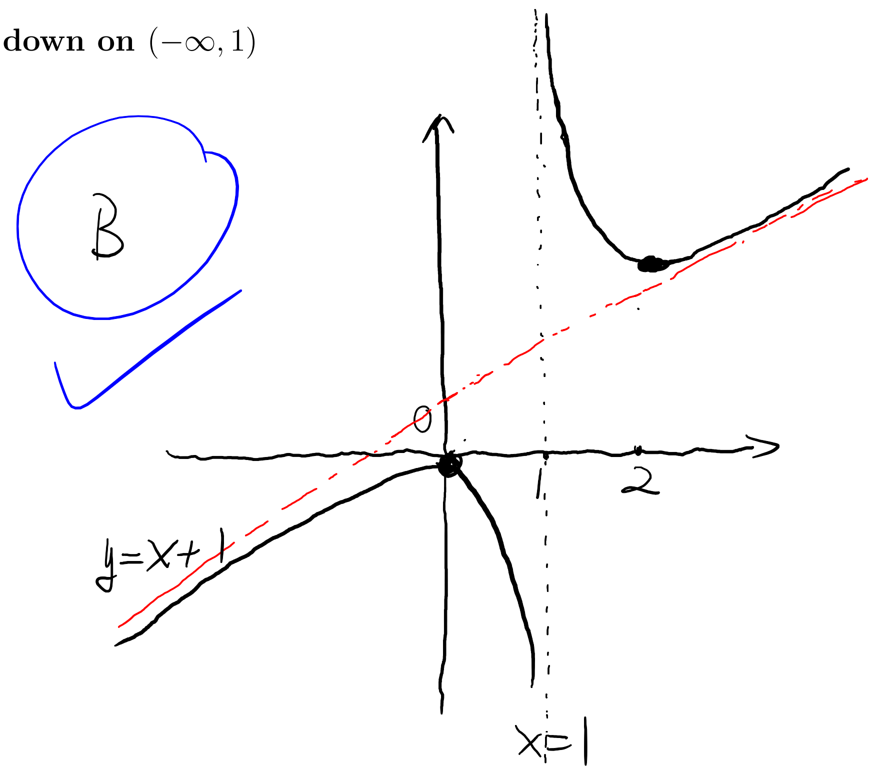
$= \frac{(x+1)(x-1)+1}{x-1}$

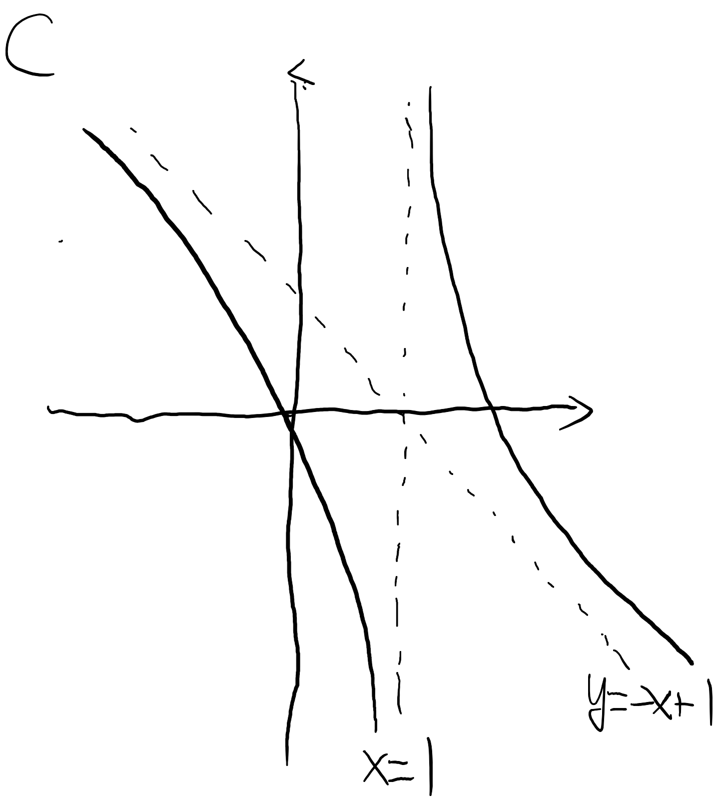
$= x+1+\frac{1}{x-1}$

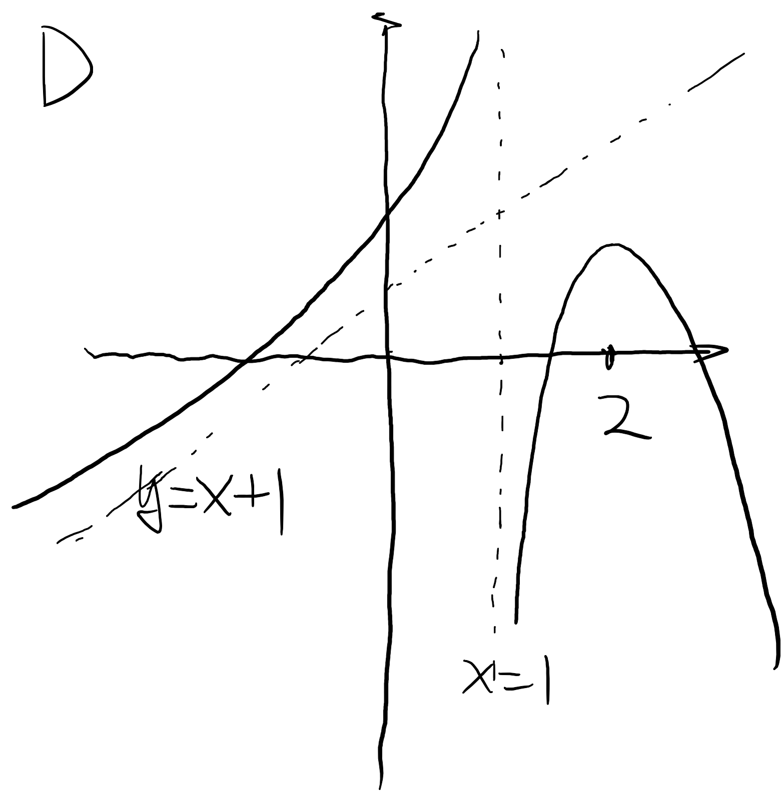
(★ 0 points) Continue with the function $f(x) = \frac{x^2}{x-1}$ in problem 2. Suppose we know the following information for $f(x)$. Which picture below best fits the graph of $y = f(x)$.

- **$f(x)$ has vertical asymptote $x = 1$. The domain of f is $(-\infty, 1) \cup (1, \infty)$**
- **f is increasing on $(-\infty, 0) \cup (2, \infty)$ and decreasing on $(0, 1) \cup (1, 2)$.**
- **f is concave up on $(1, \infty)$ and concave down on $(-\infty, 1)$**









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