Name: ______________________ **ID:** ______________

Clear your desk of everything except pens, pencils and erasers. Show all work clearly and in order. No notes, phones and calculators. You have 10 minutes to finish the test for 10 points.

ONLY THE PROBLEMS ON THIS PAGE COUNT. The problems on the back are not required, but will be graded if you finish them.

Derivative formulas:

$$(fg)' = f'g + fg',\ (\frac{f}{g})', = \frac{f'g - fg'}{g^2},\ (\sin x)' = \cos x,\ (\cos x)' = -\sin x,\ (\tan x)' = \sec^2 x,\ (\sec x)' = \sec x \cdot \tan x$$

Remark: You can use chain-rule $(f(g(x)))' = f'(g(x)) \cdot g'(x)$ **if you think necessary.**

1. **Compute the derivatives** $f'(x)$ **for the following functions** $f(x)$ **(You DO NOT need to simplify your answer after product or quotient rule):**

   (a) **(3 points)** $f(x) = (\frac{2}{x} - \sin x)(1 - 3x)$ product rule.

$$f'(x) = \left[\left(\frac{2}{x} - \sin x\right)\cdot(1-3x)\right]'$$
$$= \left(\frac{2}{x} - \sin x\right)'\cdot(1-3x) + \left(\frac{2}{x} - \sin x\right)\cdot(1-3x)'$$
$$= \boxed{(-2x^{-2} - \cos x)\cdot(1-3x) + \left(\frac{2}{x} - \sin x\right)\cdot(-3)}$$

Note: $\left(\frac{2}{x}\right)' = \left(2\cdot\frac{1}{x}\right)' = (2\cdot x^{-1})' = 2\cdot(-1)\cdot x^{-2} = -2\cdot x^{-2} = \frac{-2}{x^2}$

   (b) **(3 points)** $f(x) = \frac{2}{\cos x}$

$$f'(x) = \left(\frac{2}{\cos x}\right)'$$
$$= \frac{2'\cdot\cos x - 2\cdot(\cos x)'}{\cos^2 x}$$
$$= \frac{0\cdot\cos x - 2\cdot(-\sin x)}{\cos^2 x}$$
$$= \boxed{\frac{2\cdot\sin x}{\cos^2 x}}$$

quotient rule.

Solution 2:

$\frac{2}{\cos x} = 2\cdot\sec x.$

$$\left(\frac{2}{\cos x}\right)' = (2\sec x)'$$
$$= 2(\sec x)'$$
$$= 2\tan x\cdot\sec x$$

Direct Trig-formula.

Solution 3: chain rule

$\frac{2}{\cos x}$ outer: $\frac{2}{\blacksquare} = 2\cdot\blacksquare^{-1}$

inner: $\cos x$.

$$\left(\frac{2}{\cos x}\right)' = \underbrace{2(-1)[\cos x]^{-2}}_{\text{out}'(\text{inn})}\cdot\underbrace{(-\sin x)}_{\text{inn}'}$$
$$= 2(\cos x)^{-2}\cdot\sin x$$

2. **(4 points) Let** $h(t) = \Big(2F(t)+1\Big)\cdot G(t)$. **Let** $F(2) = -2, F'(2) = 1, G(2) = 0, G'(2) = -1$. **Calculate** $h'(2)$

Apply product rule to $h(t)$ with $f(t) = 2F(t)+1$ and $g(t) = G(t)$.

$$h'(t) = \Big[\underbrace{(2F(t)+1)}_{f}\cdot\underbrace{G(t)}_{g}\Big]' = (2F(t)+1)'\cdot G(t) + (2F(t)+1)\cdot G'(t) \quad \text{product rule.}$$
$$= (2\cdot F'(t) + 0)\cdot G(t) + (2F(t)+1)\cdot G'(t) \quad \text{linear rule}$$
$$= 2F'(t)\cdot G(t) + (2F(t)+1)\cdot G'(t) \quad \text{simplify}$$

Plug in $t=2$, $h'(2) = 2\cdot F'(2)\cdot G(2) + (2\cdot F(2)+1)\cdot G'(2)$ plug in

$$= 2\cdot 1\cdot 0 + (2\cdot(-2)+1)\cdot(-1) = 0 + (-4+1)\cdot(-1) = (-3)\cdot(-1) = \boxed{3}$$

★ 0 points

Rmk1: Point-slope formula for line passing through point (a, b) with slope k:

$$y - b = k\cdot(x-a)$$

$$\Leftrightarrow \quad y = k\cdot(x-a)+b.$$

- **Find an equation for the tangent line to the graph of**

$$y = \tan x \qquad \text{at} \qquad x = \frac{\pi}{6}$$

function: $y = \tan x$.

The line passes through $\boxed{\left(\frac{\pi}{6}, \tan\frac{\pi}{6}\right) \text{ ... point}}$

derivative: $y' = (\tan x)' = \sec^2 x$.

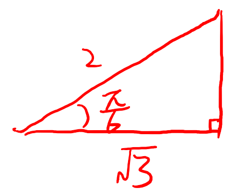
evaluate at $x = \frac{\pi}{6}$, $\boxed{\sec^2\frac{\pi}{6} \text{ ...... slope}}$.

Point-slope formula:

$$y = \sec^2\frac{\pi}{6}\cdot\left(x-\frac{\pi}{6}\right) + \tan\frac{\pi}{6} = \left(\frac{2}{\sqrt{3}}\right)^2\left(x-\frac{\pi}{6}\right)+\frac{1}{\sqrt{3}} = \boxed{\frac{4}{3}\left(x-\frac{\pi}{6}\right)+\frac{1}{\sqrt{3}}}$$

Rmk2:

$$\tan\frac{\pi}{6} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$



$$\sec\frac{\pi}{6} = \frac{2}{\sqrt{3}}$$

- **Let**

$$f(x) = \frac{2+x^2}{\sqrt{x}}$$

Find $f'(x)$.

Hint: Try to use the algebra formulas $\frac{a+b}{c} = \frac{a}{c}+\frac{b}{c}$, $\frac{x^n}{x^m} = x^{n-m}$ **instead of the quotient differential rule.**

$$f = \frac{2+x^2}{\sqrt{x}} = \frac{2}{\sqrt{x}} + \frac{x^2}{\sqrt{x}}$$

$$= \frac{2}{x^{\frac{1}{2}}} + \frac{x^2}{x^{\frac{1}{2}}}$$

$$= 2\cdot x^{-\frac{1}{2}} + x^{2-\frac{1}{2}}$$

$$= 2\cdot x^{-\frac{1}{2}} + x^{\frac{3}{2}}$$

$$f'(x) = \left(2\cdot x^{-\frac{1}{2}}\right)' + \left(x^{\frac{3}{2}}\right)'$$

$$= 2\cdot\left(-\frac{1}{2}\right)x^{-\frac{1}{2}-1} + \frac{3}{2}\cdot x^{\frac{3}{2}-1}$$

$$= \boxed{-x^{-\frac{3}{2}} + \frac{3}{2}\cdot x^{\frac{1}{2}}}$$

Quotient rule:

$$f'(x) = \frac{(2+x^2)'\cdot\sqrt{x} - (2+x^2)\cdot(\sqrt{x})'}{(\sqrt{x})^2}$$

$$= \frac{(0+2x)\sqrt{x} - (2+x^2)\cdot\frac{1}{2}\cdot x^{-\frac{1}{2}}}{x}$$

$$= \frac{2x\cdot\sqrt{x} - (2+x^2)\cdot\frac{1}{2}\cdot x^{-\frac{1}{2}}}{x}.$$