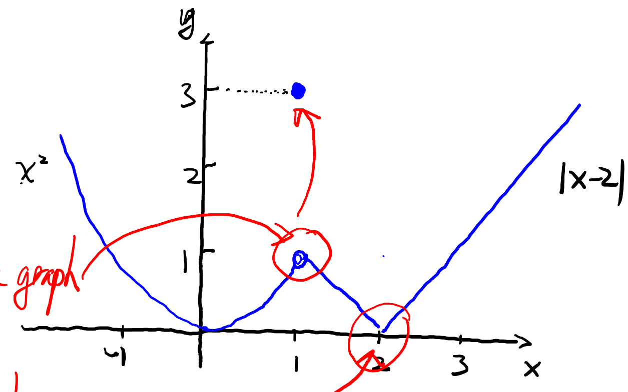
Name: ______________________ **ID:** ______________

Clear your desk of everything except pens, pencils and erasers. Show all work clearly and in order. No notes, phones and calculators. You have 10 minutes to finish the test for 10 points.

ONLY THE PROBLEMS ON THIS PAGE COUNT. The problems on the back are not required, but will be graded if you finish them.



1. Let $f(x) = \begin{cases} x^2 & x < 1 \\ 3 & x = 1 \\ |x-2| & x > 1 \end{cases}$

(a) (2 points) Is $f(x)$ continuous at $x = 1$?

f(x) is NOT continuous at x=1. A jump in the graph

(b) (2 points) Is $f(x)$ differentiable at $x = 2$?

f(x) is NOT differentiable at x=2. A sharp angle

Actually, $\lim_{h\to 0^-} \frac{f(2+h)-f(2)}{h} = -1 \neq \lim_{h\to 0^+} \frac{f(2+h)-f(2)}{h} = +1$.

(c) (2 points) Write all the interval(s) where $f(x)$ is continuous.

f(x) is continuous on $(-\infty, 1) \cup (1, +\infty)$

2. (4 points) Let $f(x) = \frac{1}{2+x}$ use <u>THE DEFINITION OF DERIVATIVE</u> to find the derivative function $f'(x)$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{-1}{(2+x+h)(2+x)} = \frac{-1}{(2+x)(2+x)} = \boxed{\frac{-1}{(2+x)^2}}$$

Caution: The limit is taken with respect to h (NOT x). Replace h by 0 (NOT x by 0).

$$f(x) = \frac{1}{2+x}$$

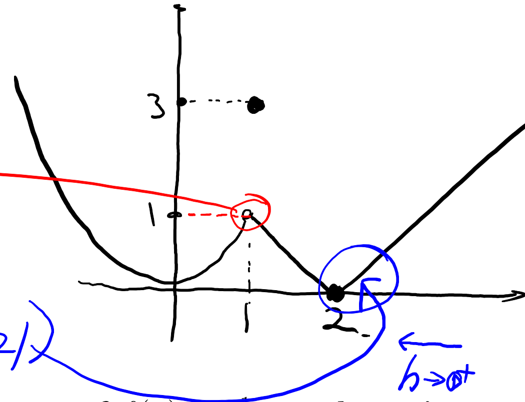
$$\Downarrow$$

$$f(x+h) = \frac{1}{2+(x+h)} \quad \text{(Plug in } x+h\text{)}$$

$$\Rightarrow f(x+h) - f(x) = \frac{1}{2+x+h} - \frac{1}{2+x} = \frac{2+x-(2+x+h)}{(2+x+h)(2+x)} = \frac{-h}{(2+x+h)(2+x)}$$

$$\Rightarrow \frac{f(x+h)-f(x)}{h} = \frac{\frac{-\cancel{h}}{(2+x+h)(2+x)}}{\cancel{h}} = \frac{-1}{(2+x+h)(2+x)}$$

★ 0 **points**



- **In problem 1, find**

$$\lim_{x\to 1} f(x) = 1$$

- **In problem 1, find**

$h \to 0^+$

$$\lim_{h\to 0+} \frac{f(2+h)-f(2)}{h} = 1 \text{ (Right derivative of } |x-2|)$$

- **In problem 2, find the equation of the tangent line of the curve of** $f(x) = \frac{1}{2+x}$ **at the point** $(1, \frac{1}{3})$.

$$f'(x) = \frac{-1}{(2+x)^2},$$

at the point $(\underbrace{1}_{x}, \underbrace{\tfrac{1}{3}}_{y})$, $f'(1) = \frac{-1}{(2+1)^2} = \frac{-1}{9}.$

Plug in

i.e. the slope of the tangent line at this point is $f'(1) = -\frac{1}{9}$.

Point-slope equation:

$$y - \frac{1}{3} = -\frac{1}{9}(x-1)$$

slope: $-\frac{1}{9}$, passing through $(1, \frac{1}{3})$

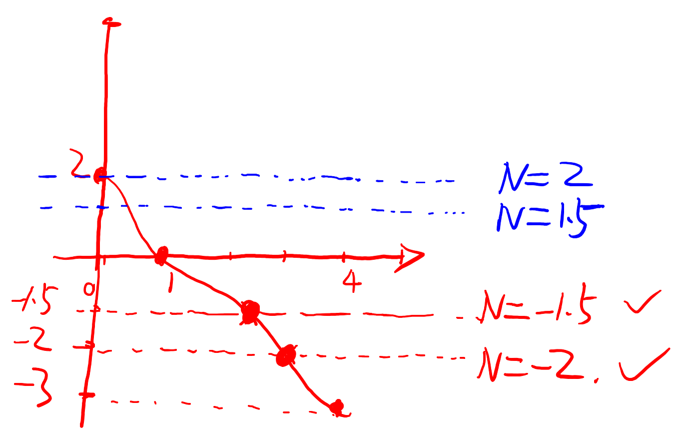
- **Suppose function** $h(x)$ **is continuous on** $[0,5]$. **Suppose** $h(0) = 2, h(1) = 0, h(4) = -3$. **For what value of** N, **the must be a** $c \in (1,4)$ **such that** $h(c) = N$? **(Hint: apply Intermediate Value Theorem)**

A $N = 1.5$

B $N = 2$

C $N = -2$ ✓

D $N = -1.5$ ✓



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