Name: ______________________ **ID:** ______________

Clear your desk of everything except pens, pencils and erasers. Show all work clearly and in order. No notes, phones and calculators. You have 10 minutes to finish the test for 10 points.

ONLY THE PROBLEMS ON THIS PAGE COUNT. The last one on the back is not required, but will be graded if you finish it.

1. **Compute the following limits (a finite number, $+\infty$ or $-\infty$)**

   (a) **(3 points)**

$$\lim_{x\to 2} \quad 3 + \frac{\sqrt{2x-1}}{x^2+2}$$

(Diret - Plug-in)

$$= 3 + \frac{\sqrt{2\cdot 2 - 1}}{2^2+2}$$

$$= \boxed{3 + \frac{\sqrt{3}}{6}}$$

   (b) **(3 points)**

$$\lim_{h\to 0} \frac{(h+2)^2-4}{h}$$

(Direct-Plug-in DOES NOT WORK, since you have $\frac{0}{0}$)

$$= \lim_{h\to 0} \frac{(h^2+4h+\cancel{4})-\cancel{4}}{h}$$

Rmk: $(h+2)^2 = (h+2)\cdot(h+2) = h^2+2h+2h+4 = h^2+4h+4$

or use $(a+b)^2 = a^2+2\cdot a\cdot b+b^2$

$$= \lim_{h\to 0} \frac{h^2+4h}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(h+4)}{\cancel{h}} = \lim_{h\to 0} h+4 \overset{\text{Plug in } 0}{=\!=\!=} \boxed{4}$$

2. **(4 points) Let $f(x) = 2-3x$ and $\varepsilon > 0$. What is the largest value of δ for which $|x-1| < \delta$ implies $|f(x)+1| < \varepsilon$:**

(e.g. 2 in Sec 1.7 lec-notes)

   A $\delta = 2\varepsilon$

   B $\delta = 3\varepsilon$

   C $\delta = \varepsilon/3$ ✓

   D None of the above

Plug $2-3x$ in $|f(x)+1| < \varepsilon$

$|2-3x+1| < \varepsilon$.

$\Leftrightarrow |3-3x| < \varepsilon$. or $\Leftrightarrow |3(1-x)| < \varepsilon$

$\Leftrightarrow -\varepsilon < 3-3x < \varepsilon$ $\quad\Leftrightarrow |1-x| < \frac{\varepsilon}{3}$

$\Leftrightarrow -\varepsilon < -3(x-1) < \varepsilon$ $\quad\Leftrightarrow |x-1| < \frac{\varepsilon}{3}$ since $|x-1| = |1-x|$

$\Leftrightarrow -\frac{\varepsilon}{3} < x-1 < \frac{\varepsilon}{3} \Leftrightarrow |x-1| < \frac{\varepsilon}{3}$.

Caution: $-3\cdot A < B$

$\Rightarrow A > \frac{B}{-3}$

(★ NOT REQUIRED. DOES NOT COUNT IN YOUR QUIZ CREDIT. The problem is of actual exam difficulty level. It is recommended to try it to check whether you handle the materials well enough for the exam.)

For what value of a does

$$\lim_{x\to 5} \frac{ax^2+25}{x-5},$$

exist (and is finite)? Find a and compute the limit.

If we plug 5 in, we have $\frac{a\cdot 25+25}{5-5} = \frac{25\cdot a+25}{0}$

The denominator is 0. The only way to ensure the limit exists (finite) is to make the numerator $25\cdot a+25$ also zero.

i.e., let $25a+25=0$

$$25a = -25 \Rightarrow \boxed{a=-1}$$

Actually, if $a=-1$, then

$$\lim_{x\to 5} \frac{a\cdot x^2+25}{x-5} = \lim_{x\to 5} \frac{-1\cdot x^2+25}{x-5}$$

Factorize:

$-x^2+25 = (-x+5)\cdot(x+5)$

and $(-x+5) = -(x-5)$

$$= \lim_{x\to 5} \frac{\cancel{(-x+5)}^{-1}\cdot(x+5)}{\cancel{x-5}}$$

$$= \lim_{x\to 5} -1\cdot(x+5)$$

Cancelling the "zero term" $\boxed{x-5}$

$\underline{\text{Plug in } 5}$ $-1\cdot(5+5)$

$$= \boxed{-10}$$