Practice Final, Sec13

Multiple Choice Problems.

1. (S17) Suppose $f(x)$ is a continuous function with values given by the table below.

x	-2	-1	0	1
f(x)	0	3	0	-3

Which of the following statement is correct?

A $f(x) = 2$ has a root $c \in (-1, 0)$.

B $f(x) = 2$ has a root $c \in (0, 1)$.

C $f(x) = 4$ has a root $c \in (-1, 0)$.

D $f(x) = 4$ has a root $c \in (-2, 1)$.

E None of the above

2. (S17) Suppose $f(x)$ is a differntiable function with values given by the table below.

x	-2	-1	0	1
f(x)	0	3	0	-3

According to Mean Value Theorem, which of the following statement is correct?

A There is $c \in (-1, 0)$ such that $f'(c) = 3$.

B There is $c \in (-2, 0)$ such that $f'(c) = 3$.

C There is $c \in (-1, 1)$ such that $f'(c) = -1$.

D There is $c \in (-2, 1)$ such that $f'(c) = -1$.

E None of the above

3. Using a linearization at $a = 100$, the linear approximation of $\sqrt{99}$ is

A $\frac{199}{20}$

B $\frac{201}{20}$

C $\frac{99}{10}$

D $\frac{101}{10}$

E None of the above

4. (S17) Suppose you are estimating the root of $x^5 = 33$ using Newton's method. If you use $x_1 = 1$, find the exact value of x_2

A $x_2 = 1 - \frac{32}{5}$

B $x_2 = 1 + \frac{32}{5}$

C $x_2 = 33 - \frac{1}{5}$

D $x_2 = 33 + \frac{32}{5}$

E $x_2 = 1 + \frac{1}{5}$

5. Evaluate the limit:

$$\lim_{x \to -3^+} \frac{x-2}{x^2(x+3)}$$

A $+\infty$

B $-\infty$

C -5

D 5

E -3

6. Find the limit:

$$\lim_{x \to \infty} \frac{x-2}{3x+5}$$

A $+\infty$

B 0

C $\frac{1}{3}$

D $-\frac{2}{3}$

E $-\frac{2}{5}$

7. Compute the limit:

$$\lim_{h \to 0} \frac{\frac{1}{h+2} - \frac{1}{2}}{h}$$

A $+\infty$

B $\frac{1}{2}$

C $\frac{1}{4}$

D $-\frac{1}{4}$

E 0

8. (Spring 16) Find the limit:

$$\lim_{x \to 1} \frac{\sin(x-1)}{x^2-1}$$

A -1

B 0

C $\frac{1}{2}$

D $-\frac{1}{1}$

E Does not exist.

9. Suppose $\int_0^2 f(x)\ dx = -4$, $\int_0^5 f(x)\ dx = 6$. Find $\int_2^5 f(x)dx$ and the average of $f(x)$ over $[2,5]$

   A $\int_2^5 f(x)dx = 2$, average of f is $\frac{2}{3}$

   B $\int_2^5 f(x)dx = 10$, average of f is $\frac{10}{3}$

   C $\int_2^5 f(x)dx = -10$, average of f is $-\frac{10}{3}$

   D $\int_2^5 f(x)dx = -2$, average of f is $-\frac{2}{3}$

   E $\int_2^5 f(x)dx = 10$, average of f is $\frac{10}{5}$

10. Evaluate

$$\int_{-\pi}^{\pi} \sin x \cdot \sqrt{\cos x + 2}\ dx$$

   A $\frac{4}{3}$

   B 0

   C $-\frac{4}{3}$

   D $-\frac{2}{3}$

   E 2

11. Suppose

$$F(x) = \int_{-\pi}^{\tan x} \sqrt{2+t^2}\ dt$$

   Find $F'(x)$

   A $\sqrt{2+(\tan x)^2}$

   B $\sqrt{2+(\tan x)^2} \cdot \sec^2 x$

   C $\sqrt{2+t^2} \cdot \sec^2 x$

   D $-\sqrt{2+\pi^2} \cdot \sec^2 x$

   E $\int_{-\pi}^{\tan x} \sqrt{2+(\tan x)^2} \cdot \sec^2 x\ dt$

12. Suppose

$$F(x) = \sqrt{2+(\tan x)^2}$$

   Find $F'(x)$

   A $\frac{1}{2}(2+(\tan x)^2)^{-1/2}$

   B $\frac{1}{2}(2+(\tan x)^2)^{-1/2} \cdot (2\tan x)$

   C $\frac{1}{2}(2+(\tan x)^2)^{-1/2} \cdot (2\tan x) \cdot (\sec^2 x)$

   D $\sqrt{2+(\tan x)^2} \cdot (2\tan x) \cdot (\sec^2 x)$

   E $\sqrt{2+(\tan x)^2} \cdot (2\tan x)$

Standard Response Problems.

1. (S17) Calculate the derivatives of $f(x) = x\sin(3x)$. And find the equation of the tangent line to the curve $y = f(x)$ at $x = \frac{\pi}{3}$

2. (S17) Suppose $f(x) = \frac{1}{x+7}$

   (a) Use the definition of the derivative to find $f'(x)$

   (b) Find the equation of the tangent line to the curve $y = f(x)$ at $x = -2$

3. (S17) Suppose that y and x satisfy the implicit equation

$$xy^3 + xy = 20$$

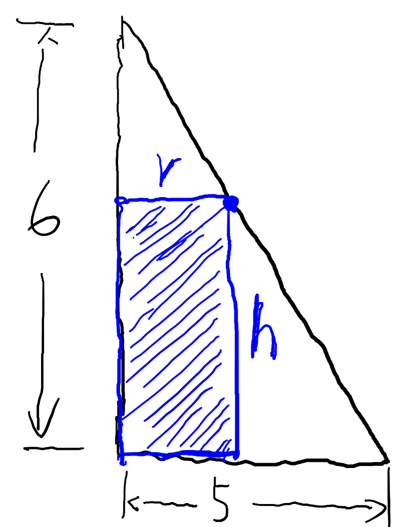
(a) Find $\frac{dy}{dx}$

(b) Use your answer in part (a) to find the equation of the tangent line to the curve $xy^3 + xy = 20$ at the point $(10, 1)$.

4. If the radius of a circular ink blot is growing at a rate of 3 cm/min. How fast (in cm^2/min) is the area of the blot growing when the radius is 10 cm?

5. Air is being pumped into a spherical balloon so that its volume increase at a rate of 100 cm^3/s. How fast is the radius of the balloon increasing when the diameter is 50cm?

6. Give a right triangle as below with base 5 cm and height 6 cm. A rectangle is inscribed with its two edges on the right triangle and its upper right corner on the hypotenuse of the right triangle. What are the dimensions of such a rectangle with the greatest possible area?



7. A particle moves with velocity $v(t) = -t^2 + 6t - 8$, $0 \leq t \leq 6$. Sketch the graph of $v(t)$ on $[0, 6]$. When is the acceleration $a(t)$ positive? When does the particle speed up?

8. (S16) Suppose $f(x) = x^4 - 6x^2 - 3$.

(a) Identify the intervals over which $f(x)$ is increasing and decreasing, and all values of x where $f(x)$ attains its local maximum or minimum.

(b) Identify the intervals over which $f(x)$ is concave up and down, and all values of x where $f(x)$ has an inflection point.

9. Calculate the integral $\int \tan^3 x \cdot \sec^2 x \; dx$

10. Calculate the integral $\int_0^{\pi/4} \tan x \cdot \sec x + 2x \; dx$

11. Find the area of the region enclosed by the graphs of the equations $y = x + 4$ and $y = x^2 - x + 1$.