- Leading term rule for limit at ∞ for Rational function

$f(x)$, $g(x)$ are linear combinations of power functions

such as $5x^4+1$, $\sqrt{x}-2x^3$, $(2x+1)\cdot(\sqrt{x}-7)$, etc.

We want to consider the limit: $\lim\limits_{x\to\infty} \dfrac{f(x)}{g(x)}$

All we need to do is to keep the $\boxed{\text{leading term}}$ in $\boxed{\text{each unit}}$, and drop all the other terms.

(lower order terms).

Rmk: Each expression in the brackets counts one unit.

e.g.

$$\frac{\overbrace{(2x^3+1)}^{\text{unit 1}}\cdot\overbrace{(3x-5)}^{\text{unit 2}}\cdot\overbrace{(x+1)}^{\text{unit 3}}}{\underbrace{(\sqrt{x}+7)}_{\text{unit 4}}\cdot\underbrace{(5x^4-7x^2)}_{\text{unit 5}}}$$

leading terms

$$\lim_{x\to\infty} \frac{(2x^3\cancel{+1})\cdot(3x\cancel{-5})\cdot(x\cancel{+1})}{(\sqrt{x}\cancel{+7})\cdot(5x^4\cancel{-7x^2})}$$

$$=\lim_{x\to\infty}\frac{2x^3\cdot 3x\cdot x}{\sqrt{x}\cdot 5x^4}=\lim_{x\to\infty}\frac{6x^5}{5\cdot x^{4.5}}$$

$$=\lim_{x\to\infty}\frac{6}{5}\cdot x^{0.5}=\infty.$$

eg. $\lim_{n\to\infty}\frac{2n^3-5n}{6n+1-7n^3}=\lim_{n\to\infty}\frac{2\cancel{n^3}}{-7\cancel{n^3}}=\frac{2}{-7}$

eg. $\lim_{n\to\infty}\frac{\sqrt{5n+1}\cdot(\sqrt{n}-2)}{2n^2-3}=\lim_{n\to\infty}\frac{\sqrt{5n}\cdot\sqrt{n}}{2n^2}=\lim_{n\to\infty}\frac{\sqrt{5}}{2n}$

$$=0$$

eg. $\lim_{n\to\infty}\frac{2n^3-\sqrt{n}}{5n^5-2n+1}\cdot\frac{5n^5}{2n^3}=\lim_{n\to\infty}\frac{(2n^3-\sqrt{n})\cdot(5n^5)}{(5n^5-2n+1)\cdot(2n^3)}$

$$=\lim_{n\to\infty}\frac{\cancel{2n^3}\cdot\cancel{5n^5}}{\cancel{5n^5}\cdot\cancel{2n^3}}=1$$