Integrals

- **Volume**: Suppose $A(x)$ is the cross-sectional area of the solid S perpendicular to the x-axis, then the volume of S is given by

Rotating Solid:

$V = \int_a^b \pi \cdot r^2 dx . (dy)$

$$V = \int_a^b A(x)\ dx$$

- **Work**: Suppose $f(x)$ is a force function. The work in moving an object from a to b is given by:

Water-Pumping:

$W = \int_a^b \sigma \cdot s(y) \cdot A(y)\, dy$

$$W = \int_a^b f(x)\ dx$$

- $\int \frac{1}{x}\ dx = \ln|x| + C$, $(\ln|x|)' = \frac{1}{x}$, $(\log_a |x|)' = \frac{1}{\ln a \cdot x}$
- $\int \tan x\ dx = \ln|\sec x| + C$, $(\tan x)' = \sec^2 x$
- $\int \sec x\ dx = \ln|\sec x + \tan x| + C$, $(\sec x)' = \tan x \cdot \sec x$
- $\int a^x\ dx = \frac{a^x}{\ln a} + C$ for $a \neq 1$ $(a^x)' = \ln a \cdot a^x$
- **Integration by Parts**:

$$\int u\ dv = uv - \int v\ du$$

$\int \underbrace{\text{Poly}}_{u} \times \underbrace{\sin/\cos/\exp}_{dv}$, $\int \underbrace{\ln/\tan^{-1}/\sin^{-1}}_{u} \cdot \underbrace{\square}_{dv}$

$\int e^{ax+b} dx = \frac{1}{a} e^{ax+b}$, $\int \sin(ax+b) dx = \frac{-1}{a} \cos(ax+b)$

$\int \cos(ax+b) dx = \frac{1}{a} \sin(ax+b)$, $\int \frac{1}{ax+b} dx = \frac{1}{a} \ln|ax+b|$

$\frac{\square}{(x-a)(x-b)} = \frac{A}{x-a} + \frac{B}{x-b}$

l'Hôpital: $\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$

Derivatives

- $\frac{d}{dx}(\sinh x) = \cosh x$ $\qquad$ $\frac{d}{dx}(\cosh x) = \sinh x$
- Inverse Trigonometric Functions:

$$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}(\csc^{-1} x) = \frac{-1}{x\sqrt{x^2-1}}$$

$$\frac{d}{dx}(\cos^{-1} x) = \frac{-1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}(\sec^{-1} x) = \frac{1}{x\sqrt{x^2-1}}$$

$$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2} \qquad \frac{d}{dx}(\cot^{-1} x) = \frac{-1}{1+x^2}$$

- If f is a one-to-one differentiable function with inverse function f^{-1} and $f'(f^{-1}(a)) \neq 0$, then the inverse function is differentiable at a and

$$(f^{-1})'(a) = \frac{1}{f'(f^{-1}(a))}$$

Hyperbolic and Trig Identities

- Hyperbolic Functions

$$\sinh(x) = \frac{e^x - e^{-x}}{2} \qquad \operatorname{csch}(x) = \frac{1}{\sinh x}$$

$$\cosh(x) = \frac{e^x + e^{-x}}{2} \qquad \operatorname{sech}(x) = \frac{1}{\cosh x}$$

$$\tanh(x) = \frac{\sinh x}{\cosh x} \qquad \coth(x) = \frac{\cosh x}{\sinh x}$$

- $\cosh^2 x - \sinh^2 x = 1$
- $\cos^2 x + \sin^2 x = 1$, $\tan^2 x + 1 = \sec^2 x$
- $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$
- $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$
- $\sin(2x) = 2 \sin x \cos x$
- $\sin A \cos B = \frac{1}{2}[\sin(A-B) + \sin(A+B)]$
- $\sin A \sin B = \frac{1}{2}[\cos(A-B) - \cos(A+B)]$
- $\cos A \cos B = \frac{1}{2}[\cos(A-B) + \cos(A+B)]$