Name: ______________________ **ID:** ______________________

Clear your desk of everything except pens, pencils and erasers. Show all work clearly and in order. No notes, phones and calculators. You have 10 minutes to finish these two problems for 10 points. (**Formula Sheet is on the back.**)

1. (6 points) Determine whether the following SERIES is convergent or divergent, state the reason (test).

- $$\sum_{n=1}^{\infty} \frac{n^2}{2n^3-1};$$

Limit Comparison Test

$a_n = \frac{n^2}{2n^3-1}$, $b_n = \frac{n^2}{2n^3} = \frac{1}{2n}$

$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{n^2}{2n^3-1} \cdot \frac{2n^3}{n^2} = \lim_{n\to\infty} \frac{2n^3}{2n^3-1} = 1 \neq 0$

Because $\sum b_n = \sum \frac{1}{2n}$ is a p-Series with $p=1 \leq 1$, $\sum b_n$ is divergent.

According to limit comparison test, $\sum a_n = \sum \frac{n^2}{2n^3-1}$ is also divergent.

- $$\sum_{n=1}^{\infty} \frac{4^n}{n!3^n}.$$

Ratio Test

$a_n = \frac{4^n}{n!\,3^n}$, $a_{n+1} = \frac{4^{n+1}}{(n+1)!\,3^{n+1}}$

$\frac{a_{n+1}}{a_n} = \frac{4^{n+1}}{(n+1)!\,3^{n+1}} \cdot \frac{n!\,3^n}{4^n} = \frac{4}{(n+1)\cdot 3}$

$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{4}{(n+1)\cdot 3} = 0 < 1.$

According to Ratio Test, $\sum a_n$ is convergent.

2. (4 points) Which statements about the series $\sum_{n=0}^{\infty} \frac{(-1)^n}{7^n}$ is true? State the reason. I. Convergent and II. Absolutely convergent

$a_n = \frac{(-1)^n}{7^n}$ $\quad |a_n| = \left|\frac{(-1)^n}{7^n}\right| = \frac{1}{7^n}$. $\sum |a_n| = \sum \frac{1}{7^n}$ is a Geometric Series with $r = \frac{1}{7} < 1$.

$\sum |a_n|$ is convergent.

Then according to the definition of absolutely convergent, $\sum a_n = \sum \frac{(-1)^n}{7^n}$ is absolutely convergent.

Therefore, both I and II are true.