Multiple Choice. Circle the best answer. No work needed. No partial credit available.

Q1 Which statement is true about the series

$$\sum_{n=1}^{\infty}(1+\frac{1}{n})^n$$

A The **nth term test** concludes that the series converges.

B The **nth term test** concludes that the series diverges.

C The **nth term test** hypotheses are not met by this series, so it cannot be applied.

D The **nth term test** hypotheses are met by this series however the test is inconclusive.

E None of the above are true. The nth term test concludes that the series converges.

$$a_n = (1+\frac{1}{n})^n = e^{\ln(1+\frac{1}{n})^n}$$

Find $\lim_{n\to\infty} \ln(1+\frac{1}{n})^n = \lim_{n\to\infty} \frac{\ln(1+\frac{1}{n})}{\frac{1}{n}}$ via l'Hospital rule first.

Q2 Which statement is true about the series

$$\sum_{n=2}^{\infty}\frac{10n}{\sqrt{n^2+2}}$$

A The **integral test** concludes that the series converges.

B The **integral test** concludes that the series diverges.

C The **integral test** hypotheses are not met by this series, so it cannot be applied.

D The **integral test** hypotheses are met by this series however the test is inconclusive.

E None of the above are true.

consider $\int_2^{\infty} \frac{10x}{\sqrt{x^2+2}}\,dx$ via u-sub $u=x^2+2$.

Q3 Determine whether the following series are absolutely convergent, conditionally convergent, or divergent:

$$(1)\ \sum_{n=1}^{\infty} \frac{\sin(n)+1}{2^n} \qquad \text{and (2)} \quad \sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n+2}}$$

A (1) is absolutely convergent; (2) is divergent.

B (1) is conditionally convergent; (2) is divergent.

C (1) is absolutely convergent; (2) is conditionally convergent.

D (1) is divergent; (2) is conditionally convergent.

E (1) and (2) are conditionally convergent.

(1). $\left|\frac{\sin(n)+1}{2^n}\right| \leq \frac{2}{2^n}$ since $-1 \leq \sin(n) \leq 1$

(2). Alternating Series with $b_n = \frac{1}{\sqrt{n+2}}$

Q4 Determine whether the following series converge or diverge.

(a)

$$\sum_{n=1}^{\infty} \frac{2^n(n^2+1)}{3^n}$$

Ratio Test

good for product/ratio of exponential $\frac{2^n}{3^n}$

and power function n^2+1

(b)

$$\sum_{n=1}^{\infty} \frac{\sqrt{n}+n^3+2n}{\sqrt{9n^8+7n}}$$

limit comparison test, choose b_n via "leading term" rule

(the terms with highest degree).

Q5 Check the convergence/divergence of

$$\sum_{n=1}^{\infty} \frac{2n+1}{n^2+n}$$

using integral test. (Note: you need to check the series satisfies ALL the THREE hypotheses of integral test.)

To check $f(n) = \frac{2n+1}{n^2+n}$ is decreasing, compute the derivative of $f(n)$ via quotient rule.

Consider $\int_1^{\infty} \frac{2x+1}{x^2+x}\,dx$ via u-sub. $u = x^2+x$.

since $du = (2x+1)\,dx$.

Q6 Find the exact arc-length of $f(x) = \frac{1}{2}x^2 - \frac{1}{4}\ln x$ from $x = 1$ to $x = 2$.

Compute $f'(x)$ first.

then complete the square using

$$\left(a + \frac{1}{4a}\right)^2 = a^2 + \left(\frac{1}{4a}\right)^2 + \frac{1}{2}.$$

to remove the square root in arclength formula.

Q7 Consider the series $2 - \frac{4}{3e} + \frac{8}{9e^2} - \frac{16}{27e^3} + \cdots$. Give the value of the nth term a_n which would allow us to rewrite this series as $\sum_{n=1}^{\infty} a_n$ and find the sum. $\longrightarrow$ Geometric.

$a = \text{first term} = 2$

$r = \dfrac{\text{2nd term}}{\text{1st term}} = \dfrac{-\frac{4}{3e}}{2}.$

then $a_n = a \cdot r^{n-1}$, $n = 1, 2, \ldots$

Q8 Find the sum of the series

$$\sum_{n=1}^{\infty} \frac{2^{2n+1} - (-1)^{n-1}}{9^n}$$

Geometric.

$$\frac{2^{2n+1}}{9^n} = \frac{2^{2n} \cdot 2}{9^n} = \frac{4^n \cdot 2}{9^n} = 2 \cdot \left(\frac{4}{9}\right)^n$$

$$= \underbrace{2 \cdot \frac{4}{9}}_{a} \cdot \underbrace{\left(\frac{4}{9}\right)}_{r}^{n-1}$$

$$\frac{(-1)^{n-1}}{9^n} = \frac{(-1)^{n-1}}{9 \cdot 9^{n-1}} = \underbrace{\frac{1}{9}}_{a} \cdot \underbrace{\left(\frac{-1}{9}\right)}_{r}^{n-1}$$

Q9 Find the radius of convergence of

$$\sum_{n=0}^{\infty} \frac{(n+3)(2x-3)^n}{3^n}$$

Apply ratio test to $a_n = \frac{(n+3)(2x-3)^n}{3^n}$.

Solve $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| < 1$, to get inequality like $|x - \frac{3}{2}| < R$

Q11 Find the first three non-zero terms of the Maclaurin series of the function

$$f(x) = xe^x + \cos x$$

use power series formulas for e^x and $\cos x$ directly.

Q12 Consider the function $f(x) = \frac{3x}{2+3x^2}$. Find the power series representation of f and the radius of convergence.

$$f(x) = 3x \cdot \frac{1}{2\left[1 + \frac{3x^2}{2}\right]}$$

$$= \frac{3x}{2} \cdot \frac{1}{1-\left(-\frac{3x^2}{2}\right)}$$

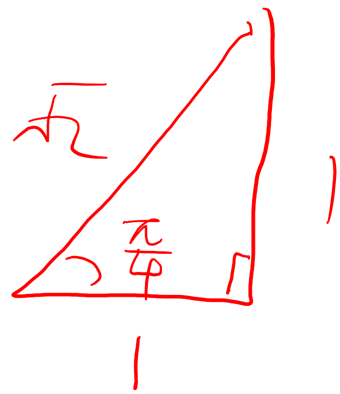
apply the corresponding formula to $\boxed{} = -\frac{3x^2}{2}$

Q13 Find the 4th degree Taylor polynomial of $f(x) = 3\sin(2x)$ centered at $a = \pi/8$

Compute the derivative table up to $n=4$

Then apply to

$$T_4(x) = f(a) + f'(a)\cdot(x-a) + \frac{f''(a)}{2}\cdot(x-a)^2 + \frac{f^{(3)}(a)}{3!}\cdot(x-a)^3 + \frac{f^{(4)}(a)}{4!}(x-a)^4.$$



$$\sin\left(\frac{\pi}{4}\right) = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}.$$

Q14 Find the Taylor series at $x = 0$ for $f(x) = x^2 e^{-2x}$ (find the general nth term and write it in Sigma notation).

Apply the formula

$$e^{\square} = \sum_{n=0}^{\infty} \frac{\square^n}{n!}. \quad \text{with } \square = -2X.$$