Practice Midterm2 Version A, Sec61

Multiple Choice. Circle the best answer. No work needed. No partial credit available.

Q1 Which statement is true about the series

$$\sum_{n=1}^{\infty} e^{\frac{2}{n}}$$

Hint: $\lim_{n} e^{\frac{2}{n}} = e^{\frac{2}{\infty}} = e^{0} = 1$

A The **nth term test** concludes that the series converges.

B The **nth term test** concludes that the series diverges.

C The **nth term test** hypotheses are not met by this series, so it cannot be applied.

D The **nth term test** hypotheses are met by this series however the test is inconclusive.

E None of the above are true. The nth term test concludes that the series converges.

Q2 Which statement is true about the series

$$\sum_{n=2}^{\infty} \frac{2\ln n}{n} \sim \int_{2}^{\infty} \frac{2\ln x}{x}\,dx$$

consider u-sub $u = \ln x$.

A The **integral test** concludes that the series converges.

B The **integral test** concludes that the series diverges.

C The **integral test** hypotheses are not met by this series, so it cannot be applied.

D The **integral test** hypotheses are met by this series however the test is inconclusive.

E None of the above are true.

Q3 Determine whether the following series are absolutely convergent, conditionally convergent, or divergent:

$$(1)\ \sum_{n=1}^{\infty} \frac{\sin(2n)}{n^2} \qquad \text{and (2)} \quad \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{3n}$$

both are webwork prob

(1). compare $\left|\frac{\sin(2n)}{n^2}\right|$ with $\frac{1}{n^2}$

A (1) is absolutely convergent; (2) is divergent.

B (1) is conditionally convergent; (2) is divergent.

C (1) is absolutely convergent; (2) is conditionally convergent.

D (1) is divergent; (2) is conditionally convergent.

E (1) and (2) are conditionally convergent.

(2). check the example for $\frac{(-1)^{n-1}}{n}$.

Q4 Determine whether the following series converge or diverge.

(a)

$$\sum_{n=1}^{\infty} \frac{\sqrt{n}+1}{e^n}$$

Ratio Test for $a_n = \frac{\sqrt{n}+1}{e^n}$

(b)

$$\sum_{n=1}^{\infty} \frac{\sqrt{n^2+n^3}}{3n^2+7n}$$

$\leftarrow$ $b_n = \frac{\sqrt{n^3}}{3n^2}$

limit Comparison test.

(c)

$$\sum_{n=1}^{\infty} \frac{n+1}{\sqrt{4n^5-1}}$$

$\leftarrow$ $b_n = \frac{n}{\sqrt{4n^5}}$

Q5 Check the convergence/divergence of

$$\sum_{n=1}^{\infty} \frac{2n}{n^2+1} = a_n = f(n)$$

using integral test. (Note: you need to check the series satisfies ALL the THREE hypotheses of integral test.)

$f(n)$ is continuous, positive.

$\boxed{\text{decreasing}}$ ⟸ Hint: compute $f'(n)$

and check $f'(n) < 0$

$$\int_1^{\infty} \frac{2x}{x^2+1}\,dx.$$

Q6 Find the exact arc-length of $f(x) = \frac{2}{3}(x^2+1)^{3/2}$ from $x = 0$ to $x = 2$.

$$\text{Arc-length} = \int_a^b \sqrt{1+[f'(x)]^2}\,dx.$$

In order to evaluate the integral, you need to complete the square via

$$1 + 4a + 4a^2 = (1+2a)^2$$

Q7 What does the series $-2 + \frac{6}{5} - \frac{18}{25} + \frac{54}{125} + \cdots$ converge to? Find the sum.

Geometric

$$= \frac{a}{1-r}$$

find a and r

a is the first term -2.

r is the common ratio $= \frac{\text{2nd term}}{\text{1st term}} = \frac{\frac{6}{5}}{-2} = -\frac{3}{5}$

Q8 Find the sum of the series

$$\sum_{n=1}^{\infty} \frac{9^{n/2}}{3(2^{2n+1})}$$

Hint: $9^{n/2} = (9^{\frac{1}{2}})^n = 3^n$

$$2^{2n+1} = 2^{2n} \cdot 2 = (2^2)^n \cdot 2 = 4^n \cdot 2$$

Q9 Find the radius of convergence of

$$\sum_{n=0}^{\infty} \frac{x^n(n^2+3)}{(-5)^n}$$

Apply ratio test to $a_n = \frac{x^n \cdot (n^2+3)}{(-5)^n}$

and solve $\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = L < 1$ for R.

where $|x| < R$

Q10 Find the first three non-zero terms of the power series representation of the function

$$f(x) = 1 - \frac{x}{1+2x^2}$$

Hint: expand $\frac{x}{1+2x^2}$, then compute

$$1 - \frac{x}{1+2x^2}$$

using

$$\frac{1}{1-\blacksquare} = \sum_{n=0}^{\infty} \blacksquare^n$$

Q11 Find the power series representation and the radius of convergence of the function

$$f(x) = \frac{x^2}{3x+2}$$

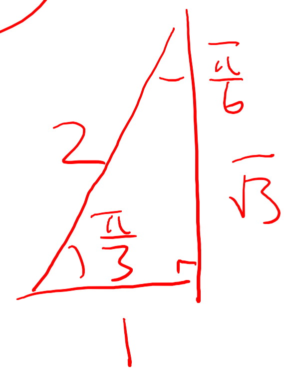
Hint: $$\frac{x^2}{3x+2} = \frac{x^2}{2\left[1+\frac{3x}{2}\right]}$$

$$= \frac{x^2}{2} \cdot \frac{1}{1-\left(-\frac{3x}{2}\right)}$$

Q12 Find the 3rd degree Taylor polynomial of $f(x) = 2 + \cos(x)$ centered at $a = \pi/3$

Derivative Table at $a = \frac{\pi}{3}$.

n	$f^{(n)}(x)$	$f^{(n)}(\frac{\pi}{3})$
$n=0$		
$n=1$		
$n=2$		
$n=3$		



$\cos\frac{\pi}{3} = \frac{1}{2}$

$\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

Q13 Find the first three non-zero terms of the Taylor series at $x = 0$ for $f(x) = 3\sin(2x) + x^2$.

use $$\sin(\square) = \sum_{n=0}^{\infty} (-1)^n \frac{\square^{2n+1}}{(2n+1)!}$$

$$= \square - \frac{\square^3}{3!} + \frac{\square^5}{5!} - \cdots$$

replace $\square$ by $2X$ and only consider the first several non-zero terms.