Practice Final Version B, Sec61

Q1 Sketch the region R bounded by $y = x^2 + 1, x = 0,\ y = 2$. Find the volume of the solid rotating R about the $y-$axis.

Q2 A vertical right-circular cylindrical tank measures 12 ft high and 10 ft in diameter. It is half full of kerosene weighing 20 lb/ft^3. Find the work it would take to pump the kerosene to the top of the tank.

Q3 Find the arc-length of the curve $x = y^{3/2}$ from $y = 0$ to $y = 2$.

Q4 Evaluate the following integrals.

(a)

$$\int 2\sin^{-1}(x)\ \mathrm{d}x$$

(b)

$$\int_0^1 xe^{2x}\ \mathrm{d}x$$

(c)

$$\int_0^{\pi/4} (\sin\theta + \cos\theta)\cos\theta\ \mathrm{d}\theta$$

Q5 Test the following improper integral. Evaluate it if it is convergent.

$$\int_0^1 \frac{1}{(x)^{4/3}}\ \mathrm{d}x$$

Q6 Determine whether each of the series is convergent or divergent.

(a)

$$\sum_{n=1}^{\infty} n \sin\left(\frac{1}{n}\right)$$

(b)

$$\sum_{n=1}^{\infty} \frac{5^n}{n!}$$

Q7 Evaluate the following limits.

(a)

$$\lim_{n\to\infty} (\ln n)^{\frac{1}{n}}$$

(b)

$$\lim_{n\to\infty} \ln\left(\frac{3n}{\sqrt{n^2+1}}\right)$$

Q8 Find the derivative of $f(x) = (\sin^{-1}(x))^{\sqrt{x}}$

Q9 Consider the following power series. Find its center and radius of convergence.

$$\sum_{n=1}^{\infty} n(2x+1)^n$$

Q10 Consider $g(t) = \frac{1}{1-t^2}$. Find the first three non-zero terms of the Maclaurin series for $g(t)$. And then find the first three non-zero terms of the Maclaurin series for $\int_0^{3x} g(t)\mathrm{d}t$.

Q11 Let $f(x) = 1/x$. Consider its Taylor series at $x = 3$.

(a) Find $T_2(x)$, the second degree Taylor polynomial of $f(x)$ centered at 3.

(b) Use Taylor's Inequality to estimate the maximum possible error in approximating $f(x)$ by $T_2(x)$ for $x \in [1, 5]$.

Q12 Solve the following differential equation

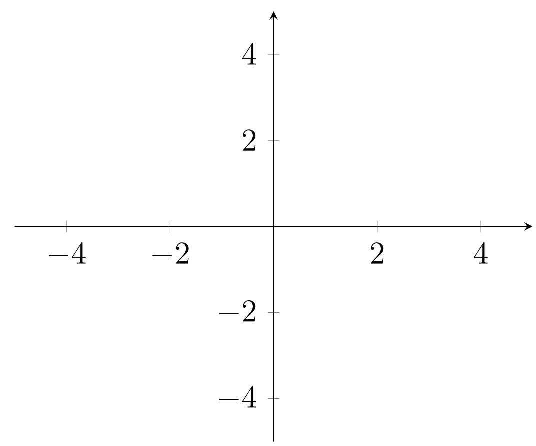
$$y'(x) = e^{-2y}x, \; y(0) = 0$$

Q13 Find the tangent line to the parametric curve

$$x(t) = \ln\Big(\sec t\Big), \quad y(t) = (t - \pi/4)^2 + 2(t - \pi/4) \qquad \text{at} \qquad t = \pi/4$$

Q14 Consider the following polar curves given by $r_1 = 2(1 + \cos\theta)$, and $r_2 = 2(1 - \cos\theta)$.

(a) Sketch both curves r_1 and r_2.



(b) Find the area of the region inside r_1 and outside r_2.