Hw Solution §8.1.

Find the exact length of the curve.

(Ed8) 9. $y = 1 + 6 \cdot x^{\frac{3}{2}}$, $\quad 0 \le x \le 1$

(Ed7) 7.

Solution: $\quad y' = 0 + 6 \cdot \frac{3}{2} \cdot x^{\frac{1}{2}} = 9x^{\frac{1}{2}}$

$$\text{Arc length} = \int_0^1 \sqrt{1 + y'^2}\, dx$$

$$= \int_0^1 \sqrt{1 + (9x^{\frac{1}{2}})^2}\, dx = \int_0^1 \sqrt{1 + 81 \cdot x}\, dx \xrightarrow[du = 81dx]{u = 1+81x} \int_1^{82} \sqrt{u}\, \frac{du}{81}$$

$$= \frac{1}{81} \cdot \frac{2}{3} \cdot u^{\frac{3}{2}} \Big|_1^{82}$$

$$= \boxed{\cancel{8}\frac{2}{243}\left(82^{\frac{3}{2}} - 1\right)}$$

(Ed8) 12. $x = \frac{y^4}{8} + \frac{1}{4y^2}$, $\quad 1 \le y \le 2$

(Ed7) 10

Solution: $\quad x' = \frac{4y^3}{8} + \frac{-2}{4y^3} = \frac{y^3}{2} - \frac{1}{2y^3}$

$$\text{Arc length} = \int_1^2 \sqrt{1 + \left(\frac{y^3}{2} - \frac{1}{2y^3}\right)^2} \cdot dy$$

$$= \int_1^2 \sqrt{1 + \left(\frac{y^3}{2}\right)^2 - 2 \cdot \frac{y^3}{2} \cdot \frac{1}{2y^3} + \left(\frac{1}{2y^3}\right)^2}\, dy$$

$$= \int_1^2 \sqrt{1 + \left(\frac{y^3}{2}\right)^2 - \frac{1}{2} + \left(\frac{1}{2y^3}\right)^2} \cdot dy$$

$$= \int_1^2 \sqrt{\left(\frac{y^3}{2}\right)^2 + \frac{1}{2} + \left(\frac{1}{2y^3}\right)^2}\, dy$$

$$= \int_1^2 \sqrt{\left(\frac{y^3}{2}\right)^2 + 2 \cdot \frac{y^3}{2} \cdot \frac{1}{2y^3} + \left(\frac{1}{2y^3}\right)^2}\, dy$$

$$= \int_1^2 \sqrt{\left(\frac{y^3}{2} + \frac{1}{2y^3}\right)^2}\, dy$$

$$= \int_1^2 \left(\frac{y^3}{2} + \frac{1}{2y^3} \right) \cdot dy$$

$$= \frac{1}{2} \cdot \frac{1}{4} y^4 + \frac{1}{2} \cdot \frac{1}{(-2)} \cdot y^{-2} \Big|_1^2$$

$$= \frac{1}{8} y^4 - \frac{1}{4} \cdot y^{-2} \Big|_1^2$$

$$= \boxed{\frac{1}{8} \cdot 2^4 - \frac{1}{4} \cdot 2^{-2} - \left(\frac{1}{8} - \frac{1}{4} \right) = \frac{33}{16}}$$

(Ed8) 14. $y = \ln(\cos x)$ $\quad 0 \le x \le \pi/3$

(Ed7) 12

$$y' = \frac{1}{\cos x} \cdot (\cos x)' = \frac{1}{\cos x} \cdot \sin x = \tan x.$$

$$\text{Arc length} = \int_0^{\pi/3} \sqrt{1 + y'^2}\, dx$$

$$= \int_0^{\pi/3} \sqrt{1 + \tan^2 x}\, dx = \int_0^{\pi/3} \sqrt{\sec^2 x}\, dx = \int_0^{\pi/3} \sec x\, dx$$

$\tan\frac{\pi}{3} = \sqrt{3}$, $\sec x = 2$

$\tan 0 = 0$, $\sec 0 = 1$

$$= \ln|\tan x + \sec x| \Big|_0^{\pi/3}$$

$$= \ln|\sqrt{3} + 2| - \ln|0 + 1|$$

$$\boxed{= \ln|\sqrt{3} + 2|}$$

Ed8) 4.

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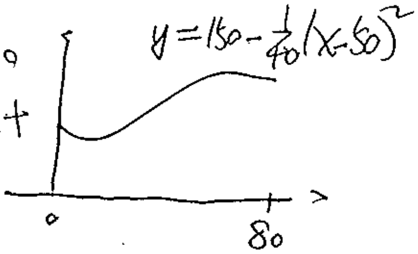
(Ed7) 36



$$y' = -\frac{1}{20}(x-50)$$

$$\text{Arc length} = \int_0^{80} \sqrt{1 + \left(\frac{x-50}{20}\right)^2}\, dx.$$

$u = \frac{x-50}{20}$

$$= \int_{-2.5}^{1.5} \sqrt{1+u^2} \cdot 20 \cdot du$$

$\cancel{u = \tan\theta}$

$\cancel{du = \sec^2\theta\, d\theta} \quad \cancel{= \int \sec\theta \cdot 20 \cdot \sec^2\theta\, d\theta}$

$\cancel{\sqrt{1+u^2} = \sqrt{1+\tan^2\theta}}$

$\cancel{= \sec\theta.} \quad \cancel{= 20 \int \sec^3\theta\, d\theta}$

Trig-Sub first

$u = \tan\theta$

$1+u^2 = \sec^2\theta$

$\sqrt{1+u^2} = \sec\theta$

$du = \sec^2\theta\, d$

$$\int \sqrt{1+u^2}\, du$$

$$= \int \sec\theta \cdot \sec^2\theta \cdot d\theta$$

$$= \int u \cdot dv$$

← Then IBP. (Example #8 textbook, P483)

$$= uv - \int v \cdot du$$

$u = \sec\theta$, $dv = \sec^2\theta\, d\theta$

$du = \tan\theta \cdot \sec\theta d\theta$ $v = \tan\theta$

$$= \sec\theta \cdot \tan\theta - \int \tan\theta\, \underline{\tan\theta \cdot \sec\theta} \cdot d\theta$$

$$= \sec\theta \cdot \tan\theta - \int (\sec^2\theta - 1) \cdot \sec\theta \cdot d\theta$$

$$= \sec\theta \cdot \tan\theta - \int \sec^3\theta - \sec\theta \cdot d\theta$$

i.e. $\int \sec^3\theta \cdot d\theta = \sec\theta \cdot \tan\theta - \int \sec^3\theta\, d\theta + \int \sec\theta \cdot d\theta$

$\Rightarrow 2\int \sec^3\theta \cdot d\theta = \sec\theta \cdot \tan\theta + \int \sec\theta \cdot d\theta$

$= \sec\theta \cdot \tan\theta + \cancel{\theta} \ln|\tan\theta + \sec\theta|$

$\Rightarrow \int \sec^3\theta \cdot d\theta = \frac{1}{2} \sec\theta \cdot \tan\theta + \frac{1}{2}\ln|\tan\theta + \sec\theta|$

$u = \tan\theta \Rightarrow \sec\theta = \sqrt{1+u^2}$

$$\int \sqrt{1+u^2}\, du = \frac{1}{2} \sqrt{1+u^2} \cdot u + \frac{1}{2} \ln|u + \sqrt{1+u^2}|$$

$\Rightarrow$ Arc length $= \int_{-2.5}^{1.5} \sqrt{1+u^2} \cdot 20 \cdot du$

$$= 20 \cdot \left(\frac{1}{2} \cdot \sqrt{1+u^2} \cdot u + \frac{1}{2} \ln|u + \sqrt{1+u^2}|\right)\Big|_{-2.5}^{1.5}$$

$$\approx 122.776$$

("Sick" problem, over 6 star, Just read it as extending materials)

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