HW §7.2 Ed 8 · Ed 7

★★ 1. $\int \sin^2 x \cdot \cos^3 x \cdot dx$ $\quad u=\sin x$, $du=\cos x \cdot dx$.

$= \int \sin^2 x \cdot \cos^2 x \cdot \underline{\cos x \cdot dx}$

$= \int u^2 \cdot \cos^2 x \cdot du$

$= \int u^2 \cdot (1-\sin^2 x)\, du = \int u^2 \cdot (1-u^2) \cdot du$

$= \int u^2 - u^4\, du = \frac{1}{3}u^3 - \frac{1}{5}u^5 + C$

$= \boxed{\frac{1}{3}(\sin x)^3 - \frac{1}{5}(\sin x)^5 + C}$

★★ 4. $\int_0^{\frac{\pi}{2}} \sin^5 x \cdot dx$ $\quad u=\cos x$. $du = -\sin x \cdot dx$

$= \int_0^{\frac{\pi}{2}} \sin^4 x \cdot \underline{\sin x \cdot dx} = \int_1 \sin^4 x \cdot (-du)$

$\sin^4 x = (\sin^2 x)^2 = (1-\cos^2 x)^2 = (1-u^2)^2$

$= \int (1-u^2)^2 (-du)$

$= \int (1-2u^2+u^4)(-du)$

$\int_{x=0}^{x=\frac{\pi}{2}} \xrightarrow{u=\cos x} \int_{u=\cos 0=1}^{u=\cos\frac{\pi}{2}=0}$

$= \int (-1+2u^2-u^4)\, du$

$= -u + 2\cdot\frac{1}{3}u^3 - \frac{1}{5}u^5 \Big|_1^0 = 0 - (-1+\frac{2}{3}-\frac{1}{5}) = \boxed{\frac{8}{15}}$

$\boxed{\text{Ed7.13}}$

★★★ 19. $\int t \sin^2 t\, dt$ $\quad$ Hint: double angle formula first

$= \int t \cdot \frac{1-\cos 2t}{2}\, dt$

$= \int \frac{t}{2} - \frac{t}{2}\cos 2t\, dt$

$= \frac{1}{2}\cdot\frac{1}{2}t^2 - \int \underbrace{\frac{t}{2}}_{u} \underbrace{\cos 2t\, dt}_{dv}$

IBP on the second integral
$u=\frac{t}{2}$; $du=\frac{1}{2}dt$,
$dv=\cos 2t\, dt$, $V=\frac{1}{2}\sin 2t$

$= \frac{1}{4}t^2 - \left[\frac{t}{2}\cdot\frac{1}{2}\sin 2t - \int \frac{1}{2}\sin 2t \cdot \frac{1}{2}dt\right]$

$= \boxed{\frac{1}{4}t^2 - \frac{t}{4}\sin 2t + \frac{1}{4}\left(-\frac{1}{2}\cos 2t\right) + C}$

☆☆ 21. $\int \tan x \cdot \sec^3 x \, dx$

Hint: tan odd, $u = \sec x$.

$du = \tan x \cdot \sec x \, dx$

$\Downarrow$

$\frac{du}{\tan x \sec x} = dx$

$$= \int \tan x \cdot \sec^3 x \cdot \frac{du}{\tan x \sec x}$$

$$= \int \sec^2 x \cdot du$$

$$= \int u^2 du = \frac{1}{3}u^3 + C = \boxed{\frac{1}{3} \cdot \sec^3 x + C}$$

☆☆☆ 25 $\int \tan^4 x \cdot \sec^6 x \, dx$.

Hint: sec even, $u = \tan x$.

$du = \sec^2 x \cdot dx$.

$\sec^4 x = (\sec^2 x)^2 = (1+\tan^2 x)^2$

$= (1+u^2)^2$

$$= \int \tan^4 x \cdot \sec^6 x \cdot \frac{du}{\sec^2 x}$$

$$= \int \underline{\tan^4 x} \cdot \underline{\sec^4 x} \cdot du$$

$$= \int u^4 (1+u^2)^2 \cdot du$$

$$= \int u^4 \cdot (1+2u^2+u^4) du = \int u^4 + 2u^6 + u^8 \, du$$

$$= \frac{1}{5}u^5 + 2 \cdot \frac{1}{7}u^7 + \frac{1}{9}u^9 + C$$

$$= \boxed{\frac{1}{5}(\tan x)^5 + \frac{2}{7}(\tan x)^7 + \frac{1}{9}(\tan x)^9 + C}$$

☆☆
☆☆ 33. $\int \underbrace{x}_{u} \cdot \underbrace{\sec x \cdot \tan x \cdot dx}_{dV}$

Hint: IBP.

$u = x$, $dV = \sec x \cdot \tan x \, dx$

$du = dx$, $V = \sec x$.

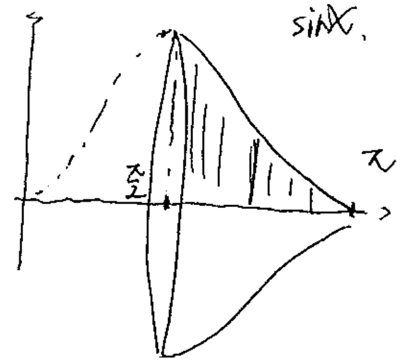
$$= x \cdot \sec x - \int \sec x \cdot dx$$

$$= \boxed{x \cdot \sec x - \ln|\tan x + \sec x| + C}$$

61. Find the volume obtained by rotating the region bounded by the curves about the given axis.

61. $y = \sin x$, $y = 0$, $\frac{\pi}{2} \le x \le \pi$, about x-axis

cross-section: Disk:



radius $= \sin x$.

$$\text{Volume} = \pi \int_{\frac{\pi}{2}}^{\pi} \sin^2 x \, dx.$$

$$= \pi \int_{\frac{\pi}{2}}^{\pi} \frac{1-\cos 2x}{2} \, dx.$$

$$= \pi \int_{\frac{\pi}{2}}^{\pi} \left(\frac{1}{2} - \frac{1}{2}\cos 2x\right) dx$$

$$= \pi \cdot \left(\frac{x}{2} - \frac{1}{2} \cdot \frac{1}{2} \sin 2x \right) \Big|_{\frac{\pi}{2}}^{\pi}$$

$$= \pi \cdot \left(\frac{\pi}{2} - \frac{1}{4} \sin 2\pi \right) - \pi \cdot \left(\frac{1}{2} \cdot \frac{\pi}{2} \right.$$