opHW7

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Sec 3.3.

10. Determine whether the vectors $\cos x$, 1, and $\sin^2(x/2)$ are linearly independent in $C[-\pi, \pi]$.

By the double angle formula:

$$\sin^2\theta = \frac{1-\cos(2\theta)}{2},$$

we have

$$\sin^2\left(\frac{x}{2}\right) = \frac{1-\cos\left(2\cdot\frac{x}{2}\right)}{2} = \frac{1}{2}\cdot 1 - \frac{1}{2}\cdot\cos x$$

Therefore

$$\frac{1}{2}\cdot 1 - \frac{1}{2}\cdot\cos x - \sin^2\left(\frac{x}{2}\right) = 0$$

They are linearly dependent.

Sec 3.4. 7, 13.

7. Find a basis for the subspace S of $\mathbb{R}^4$ consisting of all vectors of the form $(a+b, a-b+2c, b, c)^T$, where a, b, and c are all real numbers. What is the dimension of S?

$$\begin{pmatrix} a+b \\ a-b+2c \\ b \\ \end{pmatrix} = a\cdot\begin{pmatrix} 1 \\ 1 \\ \end{pmatrix} + b\cdot\begin{pmatrix} 1 \\ -1 \\ \end{pmatrix} + c\cdot\begin{pmatrix} 0 \\ 2 \\ \end{pmatrix}$$

$$\begin{pmatrix} a-b+2c \\ b \\ c \end{pmatrix} = a\cdot\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + b\cdot\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} + c\cdot\begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$$

Let. $A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & -1 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

the reduced row echelon form $\longrightarrow \begin{bmatrix} 1 & 1 & 0 \\ 0 & -2 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$\longrightarrow \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\longrightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

Therefore, $A\bar{x} = \bar{0}$ only has zero solution.

$\begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 0 \\ 1 \end{bmatrix}$ are linearly independant

and form a basis for the subspace

13. In $C[-\pi, \pi]$, find the dimension of the subspace spanned by 1, $\cos 2x$, $\cos^2 x$.

1 and $\cos 2x$ are linearly independent

By double angle formula

$\cos^2 x = \frac{1+\cos 2x}{2}$ is a linear combination of 1 and $\cos 2x$.

Therefore, $1, \cos 2x$ form a basis of $\text{span}(1, \cos 2x, \cos^2 x)$

The dimension is 2.