opHW4-2

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Sec 1.5, Optional ones: 17*, 18*

来自 <https://users.math.msu.edu/users/zhangshiwen/s19/homework.html>

17. Let A and B be $n \times n$ matrices and let $C = A - B$. Show that if $A\mathbf{x}_0 = B\mathbf{x}_0$ and $\mathbf{x}_0 \neq \mathbf{0}$, then C must be singular.

By

Theorem 1.5.2 Equivalent Conditions for Nonsingularity

Let A be an $n \times n$ matrix. The following are equivalent:

(a) *A is nonsingular.*

(b) *$A\mathbf{x} = \mathbf{0}$ has only the trivial solution $\mathbf{0}$.*

(c) *A is row equivalent to I.*

part (a), (b).

$$A x_0 = B x_0 \Rightarrow A x_0 - B x_0 = 0$$

$$\Rightarrow (A-B) \cdot x_0 = 0$$

$$\Rightarrow C \cdot x_0 = 0, \quad x_0 \neq 0$$

Therefore, $Cx = 0$ has a non-trivial solution.

By Thm 1.5.2 (a)(b), C is singular.

18. Let A and B be $n \times n$ matrices and let $C = AB$. Prove that if B is singular then C must be singular. *Hint*: Use Theorem 1.5.2.

Hint: Use Theorem 1.5.2.

B is singular, then by Thm 1.5.2 (b), there is a non-trivial vector $\bar{x} \neq \bar{0}$, such that

$$B\bar{x} = 0$$

Therefore, $C\bar{x} = (AB)\bar{x} = A\cdot(B\bar{x})$
$= A\cdot\bar{0} = \bar{0}$.

By Thm 1.5.2 again, C is singular.