Sec1.3OpHw

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9. Let

$$A = \begin{pmatrix} 1 & 2 \\ 1 & -2 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 4 \\ 0 \end{pmatrix}, \quad \mathbf{c} = \begin{pmatrix} -3 \\ -2 \end{pmatrix}$$

(a) Write **b** as a linear combination of the column vectors $\mathbf{a}_1$ and $\mathbf{a}_2$.

(b) Use the result from part (a) to determine a solution of the linear system $A\mathbf{x} = \mathbf{b}$. Does the system have any other solutions? Explain.

(c) Write **c** as a linear combination of the column vectors $\mathbf{a}_1$ and $\mathbf{a}_2$.

(a) (By observation)

$\bar{a}_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$, $\bar{a}_2 = \begin{pmatrix} 2 \\ -2 \end{pmatrix}$

$2\bar{a}_1 + \bar{a}_2 = 2\begin{pmatrix} 1 \\ 1 \end{pmatrix} + \begin{pmatrix} 2 \\ -2 \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \end{pmatrix} = \bar{b}$

(b). Let $\bar{X} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$, i.e., $x_1 = 2$, $x_2 = 1$

Then $A\bar{X} = (\bar{a}_1, \bar{a}_2) \cdot \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$

$= x_1 \cdot \bar{a}_1 + x_2 \cdot \bar{a}_2 = \bar{b}$

Therefore, $\bar{X} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ is a solution to the system.

the system.

The solution is unique since

$$A = \begin{bmatrix} 1 & 2 \\ 1 & -2 \end{bmatrix} \xrightarrow[\text{(row operation)}]{} \begin{bmatrix} 1 & 2 \\ 0 & -4 \end{bmatrix}$$ has

strict triangular form. (the explaination is not unique)

(c). Solve the system $A\bar{x} = \bar{c}$ by row operation.

$$\left[\begin{array}{cc|c} 1 & 2 & -3 \\ 1 & -2 & -2 \end{array}\right] \rightarrow \left[\begin{array}{cc|c} 1 & 2 & -3 \\ 0 & -4 & 1 \end{array}\right]$$

$\Rightarrow -4x_2 = 1 \Rightarrow x_2 = -\frac{1}{4}$

(back-sub)

$x_1 + 2\cdot(-\frac{1}{4}) = -3 \Rightarrow x_1 = \frac{1}{2} - 3 = -\frac{5}{2}$.

Therefore, $\bar{c} = \begin{pmatrix} -3 \\ -2 \end{pmatrix}$ can be written as

$$\boxed{\bar{c} = -\frac{5}{2}\bar{a}_1 - \frac{1}{4}\bar{a}_2}$$

16. A matrix A is said to be *skew symmetric* if $A^T = -A$. Show that if a matrix is skew symmetric, then its diagonal entries must all be 0.

Consider a $n \times n$ matrix $A = (a_{ij})$

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{pmatrix}$$

Then $A^T = (a_{ji}) = \begin{pmatrix} a_{11} & a_{21} & \cdots & a_{n1} \\ a_{12} & a_{22} & \cdots & a_{n2} \\ \vdots & & \ddots & \\ a_{1n} & & & a_{nn} \end{pmatrix}$

Skew symmetric means

$A = -A^T$, i.e., $(a_{ij}) = (-a_{ji})$.

In particular, on the diagonal,

we have $\boxed{a_{ii} = -a_{ii}}$ for all $i = 1, 2, \cdots, n$.

This implies $\boxed{a_{ii} = 0}$ for all $i = 1, \cdots, n$.

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