HW10

2019年4月11日 17:23

- **Sec 5.1, 1, 2 (a)(b), 3(a)(c), 17.**
- **Sec 5.2, 1 (a)(b), find N(A) and N(A^T) only; 2(a)**

来自 <https://users.math.msu.edu/users/zhangshiwen/s19/homework.html>

12 **1.** Find the angle between the vectors **v** and **w** in each of the following:

(a) $\mathbf{v} = (2, 1, 3)^T$, $\mathbf{w} = (6, 3, 9)^T$

(b) $\mathbf{v} = (2, -3)^T$, $\mathbf{w} = (3, 2)^T$

(c) $\mathbf{v} = (4, 1)^T$, $\mathbf{w} = (3, 2)^T$

(d) $\mathbf{v} = (-2, 3, 1)^T$, $\mathbf{w} = (1, 2, 4)^T$

3 (a). $V^T \cdot W = (2, 1, 3) \cdot \begin{pmatrix} 6 \\ 3 \\ 9 \end{pmatrix} = 12 + 3 + 27 = 42$

$\|V\| = \sqrt{4+1+9} = \sqrt{14}$.

$\|W\| = \sqrt{6^2+3^2+9^2} = \sqrt{36+9+81}$

$= \sqrt{9\cdot(4+1+9)} = 3\cdot\sqrt{14}$.

$\cos\theta = \dfrac{V^T \cdot W}{\|V\|\cdot\|W\|} = \dfrac{42}{\sqrt{14}\cdot 3\sqrt{14}} = \dfrac{42}{14\cdot 3} = 1$ 3pt/3

$\theta = 0$.

Actually, $w = 3v$. w and v are parallel.

Therefore the angle between the

w and v are parallel.

Therefore, the angle between them is 0.

3(b). $v^T \cdot w = (2, -3) \cdot \begin{pmatrix} 3 \\ 2 \end{pmatrix} = 6 - 6 = 0$

$\Rightarrow \cos\theta = \dfrac{v^T \cdot w}{\|v\| \cdot \|w\|} = 0 \Rightarrow \theta = \dfrac{\pi}{2}$.

3pt/3

3(c). $w^T \cdot w = (4, 1) \cdot \begin{pmatrix} 3 \\ 2 \end{pmatrix} = 12 + 2 = 14$

$\|v\| = \sqrt{16+1} = \sqrt{17}$

$\|w\| = \sqrt{9+4} = \sqrt{13}$.

$\cos\theta = \dfrac{14}{\sqrt{17} \cdot \sqrt{13}}$. $\quad \theta = \arccos\left(\dfrac{14}{\sqrt{17} \cdot \sqrt{13}}\right)$

3pt/3

3(d) $v^T \cdot w = (-2, 3, 1) \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix} = -2 + 6 + 4 = 8$

$\|v\| = \sqrt{4+9+1} = \sqrt{14}$.

$\|w\| = \sqrt{1+4+16} = \sqrt{21}$

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$\cos\theta = \dfrac{8}{\sqrt{14}\cdot\sqrt{21}} = \dfrac{8}{7\sqrt{6}}$. $\quad \theta = \arccos\left(\dfrac{8}{7\sqrt{6}}\right)$.

3pt/3

8 **2.** For each pair of vectors in Exercise 1, find the scalar projection of **v** onto **w**. Also find the vector projection of **v** onto **w**.

(a) $\mathbf{v} = (2, 1, 3)^T$, $\mathbf{w} = (6, 3, 9)^T$

(b) $\mathbf{v} = (2, -3)^T$, $\mathbf{w} = (3, 2)^T$

4 (a). scalar projection:

$\alpha = \|v\| \cdot \cos\theta = \|v\| = \sqrt{14}$ 2pt/4

vector projection:

$\bar{p} = \alpha \cdot \bar{u}$, where $\bar{u}$ is the unit vector of w.

$\bar{u} = \dfrac{1}{\|w\|} \cdot w = \dfrac{1}{3\sqrt{14}} \cdot (6, 3, 9)^T$

$= \dfrac{1}{\sqrt{14}}(2, 1, 3)^T$

[illegible]

$$\bar{p} = \sqrt{14} \cdot \frac{1}{\sqrt{14}} \cdot (2,1,3)^T = (2,1,3)^T$$ 2pt/4.

Actually, $\bar{p} = v$.

4. (b). scalar projection:

$$\alpha = \|v\| \cdot \cos\theta = 0.$$ 2pt/4

vector projection:

$$\bar{p} = \alpha \cdot \bar{u} = (0,0,0)^T$$ 2pt/4

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3. For each of the following pairs of vectors **x** and **y**, find the vector projection **p** of **x** onto **y** and verify that **p** and **x** − **p** are orthogonal:

(a) $\mathbf{x} = (3,4)^T$, $\mathbf{y} = (1,0)^T$

(b) $\mathbf{x} = (3,5)^T$, $\mathbf{y} = (1,1)^T$

(c) $\mathbf{x} = (2,4,3)^T$, $\mathbf{y} = (1,1,1)^T$

6 (a). scalar projection

$$\alpha = \|x\| \cdot \cos\theta$$

$$= \|x\| \cdot \frac{x^T \cdot y}{\|x\| \cdot \|y\|} = \frac{x^T \cdot y}{\|y\|}$$

$= \|x\| \cdot \frac{x \cdot y}{\|x\|\|y\|} = \frac{x \cdot y}{\|y\|}$

$= \frac{3+0}{\sqrt{1^2+0^2}} = 3.$

unit vector of $\bar{y}$

$\bar{u} = \frac{1}{\|\bar{y}\|} \bar{y} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}.$

vector projection:

$\bar{p} = \alpha \cdot \bar{u} = 3 \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 3 \\ 0 \end{pmatrix}.$ 3pt/6

$\bar{x} - \bar{p} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} - \begin{pmatrix} 3 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 4 \end{pmatrix}.$ 1pt/6

Then $\bar{p}^T \cdot (\bar{x} - \bar{p}) = (3, 0) \cdot \begin{pmatrix} 0 \\ 4 \end{pmatrix} = 0$ 2pt/6

$\bar{p}$ and $\bar{x} - \bar{p}$ are orthogonal.

6 (c). $\bar{x} = \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix}, \quad \bar{y} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}.$

$\alpha = \|x\| \cdot \cos\theta = \frac{\bar{x}^T \cdot \bar{y}}{\|\bar{y}\|}$

$\|\bar{y}\|$

$$= \frac{2+3+4}{\sqrt{3}} = \frac{9}{\sqrt{3}} = 3\sqrt{3}$$

unit vector of $\bar{y}$:

$$\bar{u} = \frac{1}{\|\bar{y}\|} \cdot \bar{y} = \frac{1}{\sqrt{3}} \cdot \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

vector projection:

$$\bar{p} = \alpha \cdot \bar{u} = 3\sqrt{3} \cdot \frac{1}{\sqrt{3}} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \\ 3 \end{pmatrix}$$ 3pt/6

$$\bar{x} - \bar{p} = \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix} - \begin{pmatrix} 3 \\ 3 \\ 3 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}.$$ 1pt/6

$$\bar{p}^T \cdot (\bar{x} - \bar{p}) = (3, 3, 3) \cdot \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} = -3+0+3 = 0$$ 2pt/6.

$\bar{p}$ and $\bar{x} - \bar{p}$ are orthogonal

Sec 5.2, 1 (a)(b), find N(A) and N(A^T) only; 2(a)

来自 <https://users.math.msu.edu/users/zhangshiwen/s19/homework.html>

12 **1.** For each of the following matrices, determine a basis for each of the subspaces $R(A^T)$, $N(A)$, $R(A)$

12 **1.** For each of the following matrices, determine a basis for each of the subspaces $R(A^T)$, $N(A)$, $R(A)$, and $N(A^T)$:

(a) $A = \begin{bmatrix} 3 & 4 \\ 6 & 8 \end{bmatrix}$ **(b)** $A = \begin{bmatrix} 1 & 3 & 1 \\ 2 & 4 & 0 \end{bmatrix}$

6 (a). $A\bar{x} = \bar{0}$

$$\left[\begin{array}{cc|c} 3 & 4 & 0 \\ 6 & 8 & 0 \end{array}\right] \rightarrow \left[\begin{array}{cc|c} 3 & 4 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

$3x_1 + 4x_2 = 0$ $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \alpha \cdot \begin{pmatrix} -\frac{4}{3} \\ 1 \end{pmatrix}$.

$N(A) = \left\{ \alpha \cdot \begin{pmatrix} -\frac{4}{3} \\ 1 \end{pmatrix} \middle| \alpha \in \mathbb{R} \right\}$ 3pt/6

or $\left\{ \alpha \cdot \begin{pmatrix} -4 \\ 3 \end{pmatrix} \middle| \alpha \in \mathbb{R} \right\}$.

$A^T \bar{x} = \bar{0}$.

$$\left[\begin{array}{cc|c} 3 & 6 & 0 \\ 4 & 8 & 0 \end{array}\right] \rightarrow \left[\begin{array}{cc|c} 1 & 2 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

$$x_1 + 2x_2 = 0 \Rightarrow \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \alpha \cdot \begin{pmatrix} -2 \\ 1 \end{pmatrix}.$$

$$N(A^T) = \left\{ \alpha \cdot \begin{pmatrix} -2 \\ 1 \end{pmatrix} \,\middle|\, \alpha \in \mathbb{R} \right\}.$$ 3pt/6.

6 (b) $A\overline{x} = \overline{0}$.

$$\left[\begin{array}{ccc|c} 1 & 3 & 1 & 0 \\ 2 & 4 & 0 & 0 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 3 & 1 & 0 \\ 0 & -2 & -2 & 0 \end{array}\right]$$

$$\to \left[\begin{array}{ccc|c} 1 & 3 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{array}\right]$$

$$\to \left[\begin{array}{ccc|c} 1 & 0 & -2 & 0 \\ 0 & 1 & 1 & 0 \end{array}\right]$$

$$\begin{cases} x_1 - 2x_3 = 0 \\ x_2 + x_3 = 0 \end{cases}, \quad x_3 = \alpha. \quad \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \alpha \cdot \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$$

$$N(A) = \left\{ \alpha \cdot \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} \,\middle|\, \alpha \in \mathbb{R} \right\}$$ 3pt/6

$$A^T \cdot \overline{x} = \overline{0}$$

$A^T \cdot \overline{x} = \overline{0}$

$$\left(\begin{array}{cc|c} 1 & 2 & 0 \\ 3 & 4 & 0 \\ 1 & 0 & 0 \end{array}\right) \to \left(\begin{array}{cc|c} 1 & 2 & 0 \\ 0 & -2 & 0 \\ 0 & -2 & 0 \end{array}\right)$$

$$\to \left(\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right)$$

$x_1 = 0, \quad x_2 = 0.$

$x_3 = \alpha.$ (free)

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \alpha \cdot \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \quad N(A^T) = \left\{ \alpha \cdot \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \middle| \alpha \in \mathbb{R} \right\}$$

3pt/6

5 **2.** Let S be the subspace of $\mathbb{R}^3$ spanned by $\mathbf{x} = (1, -1, 1)^T$.

(a) Find a basis for $S^\perp$.

It is enough to find all $\overline{y} \in \mathbb{R}^3$

such that

$$(1, -1, 1) \cdot \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = 0$$

2pt

$\Leftrightarrow \quad y_1 - y_2 + y_3 = 0$

$y_1 = y_2 - y_3,$

$= \alpha - \beta, \quad y_2 = \alpha, \; y_3 = \beta$

$$\begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} \alpha - \beta \\ \alpha \\ \beta \end{pmatrix} = \alpha \cdot \underbrace{\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}}_{1pt/5} + \beta \cdot \underbrace{\begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}}_{1pt/5}$$

$$S^{\perp} = \left\{ \alpha \cdot \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + \beta \cdot \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \;\middle|\; \alpha, \beta \in \mathbb{R} \right\}$$

$\underbrace{\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \text{ form a basis of } S^{\perp}.}_{1pt/5}$