

Name: \_\_\_\_\_

ID: \_\_\_\_\_

Clear your desk of everything except pens, pencils and erasers. Show all work clearly and in order. No notes, phones and calculators. You have 10 minutes to finish the test for 10 points.

The one on the back worth two extra points. Maximum of 10 points will be recorded for each quiz.

1. Compute the following limits.

(a) (4 points)

$$\begin{aligned} \lim_{x \rightarrow 2^+} \frac{2-x}{x^2-4} &= \lim_{x \rightarrow 2^+} \frac{-\cancel{(x-2)}}{\cancel{(x-2)}(x+2)} \\ &= \lim_{x \rightarrow 2^+} \frac{-1}{x+2} \\ \text{Plug in} \quad \frac{-1}{2+2} &= \boxed{-\frac{1}{4}} \end{aligned}$$

(b) (3 points)

$$\begin{aligned} \lim_{h \rightarrow 0} \frac{2(1+h) + 1 - 3}{h} &= \lim_{h \rightarrow 0} \frac{\cancel{2} + 2h + \cancel{1} - \cancel{3}}{h} \\ &= \lim_{h \rightarrow 0} \frac{\cancel{2}h}{h} = \lim_{h \rightarrow 0} 2 = \boxed{2} \end{aligned}$$

2. (3 points) Let

$$f(x) = \frac{x\sqrt{x-1}}{x-2}$$

Find the Domain of Continuity of  $f(x)$ .

$$\sqrt{x-1}, \quad x-1 \geq 0 \Rightarrow x \geq 1. \quad x-2 \neq 0 \Leftrightarrow x \neq 2$$

$$\text{D.O.C. } x \geq 1 \text{ and } x \neq 2, \text{ i.e., } \boxed{[1, 2) \cup (2, +\infty)}$$

(★ Finish the problems on the front page first. No more than 10 points may be earned on the quiz. The extra problem is of average (or above) actual exam difficulty level. It is recommended to do it now or later to check whether you handle the materials well enough for the exam.)

[2 extra points] Consider the following piecewise function:

$$g(x) = \begin{cases} -\frac{1}{4} & ; \quad x \leq 2 \\ \frac{2-x}{x^2-4} & ; \quad x > 2 \end{cases}$$

Is  $g(x)$  continuous at  $x = 2$  or not? Why?

Hint: A function  $f(x)$  is said to be continuous at  $x = a$  if  $\lim_{x \rightarrow a} f(x) = f(a)$ . Use the right side limit you get in Problem 1, Part (a) for  $\frac{2-x}{x^2-4}$  to answer this question.

$$x \leq 2, \quad g(x) = -\frac{1}{4}, \Rightarrow \lim_{x \rightarrow 2^-} g(x) = -\frac{1}{4}.$$

$$x > 2, \quad g(x) = \frac{2-x}{x^2-4} \Rightarrow \lim_{x \rightarrow 2^+} g(x) = \lim_{x \rightarrow 2^+} \frac{2-x}{x^2-4} = -\frac{1}{4}. \quad (\text{L'Hôpital's rule})$$

$$\text{Therefore, } \lim_{x \rightarrow 2} g(x) = -\frac{1}{4}.$$

$$x \leq 2, \quad g(x) = -\frac{1}{4} \Rightarrow g(2) = -\frac{1}{4}.$$

$$\text{Therefore, } \boxed{\lim_{x \rightarrow 2} g(x) = -\frac{1}{4} = g(2)}$$

$g(x)$  is continuous at  $x=2$ .