Name: $\qquad$ ID: $\qquad$
Complete the problems for 20 points. The scores will be recorded as Quiz9 and Quiz10 (10 pts each).

1. (8 points) Give an isosceles triangle with base 10 cm and height 5 cm . A rectangle is inscribed with its base on the base of the isosceles triangle and its upper corners on the two legs (the two two equal sides). What are the dimensions of such a rectangle with the greatest possible area? Find the greatest possible area.

2. (2 points) Find the most general anti-derivative of

$$
f(x)=4 \cos x+8-\frac{\tan x \sec x}{3}
$$

3. (3 points) Find $y$ (as a function of $x$ ) if $y^{\prime}=\sin x$ and $y(0)=2$.
4. (4 points) Estimate the area under the graph of $f(x)=28+12 x-x^{2}$ from $x=-2$ to $x=14$. Find the overestimate of the area using the "upper sum" of the areas of 4 rectangles of equal width.
5. (3 points) Find the sum

$$
\sum_{i=0}^{2}\left(3^{i}-1\right)
$$

## Algebraic

- $a^{2}-b^{2}=(a-b)(a+b)$
- $a^{3}-b^{3}=(a-b)\left(a^{2}+a b+b^{2}\right)$
- Quadratic Formula: $\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a}$


## Geometric

- Area of Circle: $\pi r^{2}$
- Circumference of Circle: $2 \pi r$
- Circle with center $(h, k)$ and radius $r$ :

$$
(x-h)^{2}+(y-k)^{2}=r^{2}
$$

- Distance from $\left(x_{1}, y_{1}\right)$ to $\left(x_{2}, y_{2}\right)$ :

$$
\sqrt{\left(x_{1}-x_{2}\right)^{2}+\left(y_{1}-y_{2}\right)^{2}}
$$

- Area of Triangle: $\frac{1}{2} b h$
- $\sin \theta=\frac{\text { opposite leg }}{\text { hypotenuse }}$
- $\cos \theta=\frac{\text { adjacent leg }}{\text { hypotenuse }}$
- $\tan \theta=\frac{\text { opposite leg }}{\text { adjacent leg }}$
- If $\triangle A B C$ is similar to $\triangle D E F$ then

$$
\frac{A B}{D E}=\frac{B C}{E F}=\frac{A C}{D F}
$$

- Volume of Sphere: $\frac{4}{3} \pi r^{3}$
- Surface Area of Sphere: $4 \pi r^{2}$
- Volume of Cylinder/Prism: (height)(area of base)
- Volume of Cone/Pyramid: $\frac{1}{3}$ (height)(area of base)


## Trigonometric

- $\sin ^{2} \theta+\cos ^{2} \theta=1$
- $\sin (2 \theta)=2 \sin \theta \cos \theta$
- $\cos (2 \theta)=\cos ^{2} \theta-\sin ^{2} \theta$

$$
\begin{aligned}
& =1-2 \sin ^{2} \theta \\
& =2 \cos ^{2} \theta-1
\end{aligned}
$$

- Table of Trig Values

| $x$ | 0 | $\pi / 6$ | $\pi / 4$ | $\pi / 3$ | $\pi / 2$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\sin (x)$ | 0 | $1 / 2$ | $\sqrt{2} / 2$ | $\sqrt{3} / 2$ | 1 |
| $\cos (x)$ | 1 | $\sqrt{3} / 2$ | $\sqrt{2} / 2$ | $1 / 2$ | 0 |
| $\tan (x)$ | 0 | $\sqrt{3} / 3$ | 1 | $\sqrt{3}$ | DNE |

## Limits

- $\lim _{x \rightarrow a} f(x)$ exists if and only if $\lim _{x \rightarrow a^{-}} f(x)=\lim _{x \rightarrow a^{+}} f(x)$
- $\lim _{\theta \rightarrow 0} \frac{\sin \theta}{\theta}=1$
- $\lim _{\theta \rightarrow 0} \frac{1-\cos \theta}{\theta}=0$


## Derivatives

- $f^{\prime}(x)=\lim _{h \rightarrow 0} \frac{f(x+h)-f(x)}{h}$
- $(\cot x)^{\prime}=-\csc ^{2} x$
- $(\csc x)^{\prime}=-\csc x \cdot \cot x$


## Theorems

- (IVT) If $f$ is continuous on $[a, b], f(a) \neq f(b)$, and $N$ is between $f(a)$ and $f(b)$ then there exists $c \in(a, b)$ that satisfies $f(c)=N$.
- (MVT) If $f$ is continuous on $[a, b]$ and differentiable on $(a, b)$ then there exists $c \in(a, b)$ that satisfies $f^{\prime}(c)=\frac{f(b)-f(a)}{b-a}$.
- (FToC P1) If $F(x)=\int_{a}^{x} f(t) d t$ then $F^{\prime}(x)=f(x)$.


## Other Formulas

- Newton's Method: $\quad x_{n+1}=x_{n}-\frac{f\left(x_{n}\right)}{f^{\prime}\left(x_{n}\right)}$
- $\sum_{i=1}^{n} c=c n$
- $\sum_{i=1}^{n} i=\frac{n(n+1)}{2}$
- $\sum_{i=1}^{n} i^{2}=\frac{n(n+1)(2 n+1)}{6}$

