Name: ______________________ **ID:** ______________

Clear your desk of everything except pens, pencils and erasers. Show all work clearly and in order. No notes, phones and calculators. You have 10 minutes to finish the test for 10 points.

ONLY THE PROBLEMS ON THIS PAGE COUNT. The problems on the back are not required, but will be graded if you finish them.

- $$(fg)' = f'g + fg', \ \left(\frac{f}{g}\right)', = \frac{f'g - fg'}{g^2}, \quad (f(g(x)))' = f'(g(x)) \cdot g'(x)$$
- **(MVT) If f is continuous on $[a, b]$ and differentiable on (a, b) then there exists $c \in (a, b)$ that satisfies $f'(c) = \frac{f(b)-f(a)}{b-a}$**

1. **(4 points) If the Mean Value Theorem is applied to the function $f(x) = x^2 + x$ on the interval $[0, 3]$, what value of c satisfies the conclusion of the theorem in this case?**

$f'(x) = (x^2+x)' = 2x+1 \Rightarrow f'(c) = 2c+1.$

$a=0$, $b=3$. $f(0) = 0^2+0 = 0$, $f(3) = 3^2+3 = 12$

MVT: $f'(c) = \frac{f(3)-f(0)}{3-0} \Rightarrow 2c+1 = \frac{12-0}{3} = 4$

Solve for c: $2c+1=4 \Rightarrow \boxed{c = \frac{3}{2}}$.

2. **Suppose $f(x) = x^3 - 3x^2$.**
   (a) **(2 points) Compute $f'(x)$ and find all x such that $f'(x) = 0$.**
   (b) **(2 points) Identify the intervals over which $f(x)$ is increasing and decreasing.**
   (c) **(2 points) Find all values of x where $f(x)$ attains its local maximum or minimum.**

(a). $f'(x) = 3x^2 - 3\cdot 2x = 3x^2 - 6x = 0 \Rightarrow 3x(x-2) = 0$

$\Rightarrow x = 0, \ x = 2.$

(b).

x	$x<0$		$0<x<2$		$x>2$
$f'(x)=3x(x-2)$	$+++$	0	$---$	2	$+++$
	$f'(-1)>0$		$f'(1)<0$		$f'(3)>0$

f is increasing where $f'>0$, i.e., $(-\infty, 0) \cup (2, +\infty)$ and is decreasing on $(0, 2)$.

(c). At $x=0$, f' changes from $+$ to $-$, i.e., f attains local max at $x=0$

At $x=2$, f' changes from $-$ to $+$, i.e., f attains local min at $x=2$

(★ 0 points) **Continue with the function $f(x) = x^3 - 3x^2$ in problem 2.**

- **Compute $f''(x)$ and find all x such that $f''(x) = 0$.**
- **Identify the intervals over which $f(x)$ is concave up and down, and all values of x where $f(x)$ has an inflection point.**
- **Write down the x-intercepts and y-intercept for $f(x)$. Use the information in all the previous parts about $f(x), f'(x)$ and $f''(x)$, sketch the curve of $y = f(x)$.**

- $f'(x) = 3x^2 - 6x \Rightarrow f''(x) = 6x - 6 = 0 \Rightarrow x = 1$.

x	$x<1$	1	$x>1$
$f'' = 6(x-1)$.	$f''<0$		$f''>0$.

  f is concave up where $f''>0$, i.e., on $(1, +\infty)$

  f is concave down where $f''<0$, i.e., on $(-\infty, 1)$

  $x=1$ is an inflection point.

- Sketch the curve:

x intercepts:

$f = x^3 - 3x^2 = 0$

$\Rightarrow x = 0, x = 3$.

y intercept

$y = 0$.