Name: _____ **ID:** _____

Clear your desk of everything except pens, pencils and erasers. Show all work clearly and in order. No notes, phones and calculators. You have 10 minutes to finish the test for 10 points.

ONLY THE PROBLEMS ON THIS PAGE COUNT. The problems on the back are not required, but will be graded if you finish them.

- $$(fg)' = f'g + fg', \quad \left(\frac{f}{g}\right)', = \frac{f'g - fg'}{g^2}, \quad (f(g(x)))' = f'(g(x)) \cdot g'(x)$$

- **Linearization of f at a :** $L(x) = f(a) + f'(a)(x - a)$

1. **(5 points) Find the linearization of $f(x) = \frac{1}{\sqrt{x}}$ at the point $x = 4$ and use this linearization to find a good approximation of $\frac{1}{\sqrt{4.01}}$ (Do not need to simplify).**

$f'(x) = \left(\frac{1}{\sqrt{x}}\right)' = \left(x^{-\frac{1}{2}}\right)' = -\frac{1}{2} \cdot x^{-\frac{3}{2}}$, $a=4$, $f'(4) = -\frac{1}{2} \cdot 4^{-\frac{3}{2}}$

$f(4) = \frac{1}{\sqrt{4}} = \frac{1}{2}$

$L(x) = f(4) + f'(4) \cdot (x-4)$

$= \frac{1}{2} - \frac{1}{2} \cdot 4^{-\frac{3}{2}} \cdot (x-4)$.

Remark: • $f(a)$ and $f'(4)$ in the formula are constants (numbers), do not contain x

• $4^{-\frac{3}{2}}$ can be simplified as $4^{-\frac{3}{2}} = \frac{1}{4^{\frac{3}{2}}} = \frac{1}{(\sqrt{4})^3} = \frac{1}{2^3} = \frac{1}{8}$.

2. **(5 points) Find all the critical values of $f(x) = x^2(2x - 6)$ on the interval $[-2, 1]$**

$f(x) = x^2(2x-6) = 2x^3 - 6x^2$

$f'(x) = (2x^3 - 6x^2)' = 2 \cdot 3x^2 - 6 \cdot 2x = 6x^2 - 12x = 6x(x-2)$.

Set $f'(x) = 6x \cdot (x-2) = 0 \Rightarrow x = 0$, $x = 2$ (not in $[-2, 1]$)

The critical value of f in $[-2, 1]$ is $x = 0$.

Remark: • critical values (points) $x = c$ are all c such that $f'(c) = 0$.

• f' can be computed directly through product rule (longer)

$f'(x) = (x^2 \cdot (2x-6))' = (x^2)' \cdot (2x-6) + x^2 \cdot (2x-6)'$

$= 2x \cdot (2x-6) + x^2 \cdot 2 = 4x^2 - 12x + 2x^2 = 6x^2 - 12x$

★ 0 **points**

- **Find the absolute maximum and absolute minimum values of of $f(x) = x^2(2x-6)$ on the interval $[-2, 1]$, and all values of x where $f(x)$ attains its absolute maximum or minimum.**

critical point: $x=0$. $f(0)=0$ ← largest

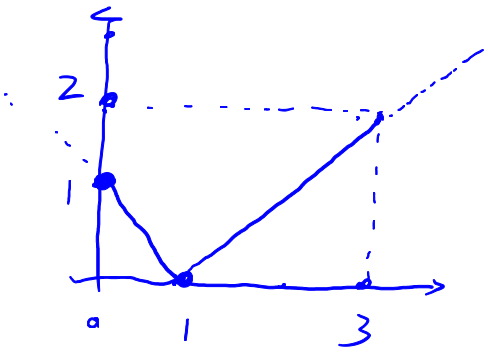
endpoints: $x=-2$, $f(-2)=(-2)^2\cdot(-4-6)=-40$ ← smallest.

$x=1$, $f(1)=1^2\cdot(2-6)=-4$.

f attains its abs maximum at 0. The abs maximum is 0.

f attains its abs minimum at -2. The abs minimum is -40

- **Find the absolute maximum and absolute minimum values of of $y = |x-1|$ on the interval $[0, 3]$. (Hint: sketch the curve of $y = |x-1|$)**



From the graph, we see that

the absolute maximum is 2, attained at 3.

the absolute minimum is 0, attained at 1.