

Name: \_\_\_\_\_

ID: \_\_\_\_\_

Clear your desk of everything except pens, pencils and erasers. Show all work clearly and in order. No notes, phones and calculators. You have 10 minutes to finish the test for 10 points.

**ONLY THE PROBLEMS ON THIS PAGE COUNT.** The one on the back is not required, but will be graded if you finish it.

1. (4 points) A particle moves according to the law of motion

$$s(t) = t^3 - 2t^2 + 3t, \quad t > 0$$

where  $t$  is measured in seconds and  $s$  in feet. Find the average velocity over the interval  $[0, 2]$ .

Apply the formula  $V_{ave} = \frac{s(t_2) - s(t_1)}{t_2 - t_1}$  on  $[0, 2]$ .

$$s(2) = 2^3 - 2 \cdot 2^2 + 3 \cdot 2 = 8 - 2 \cdot 4 + 6 = 6 \quad (\text{ft})$$

$$s(0) = 0 - 0 + 0 = 0$$

$$\text{Average velocity} = \frac{s(2) - s(0)}{2 - 0} = \frac{6 - 0}{2} = \boxed{3} \quad \text{ft/s}$$

2. Compute the following limits (a finite number,  $+\infty$  or  $-\infty$ ) where

$$f(x) = \frac{1}{x - 2}$$

- (a) (3 points)

$$\lim_{x \rightarrow 4} f(x) \quad (\text{Directly plug in 4})$$

$$= \lim_{x \rightarrow 4} \frac{1}{4 - 2} = \boxed{\frac{1}{2}}$$

- (b) (3 points)

$$\lim_{x \rightarrow 2^-} f(x) \quad \text{Plug in 2, we have } \frac{1}{0}, \text{ which suggests the answer is } \text{DNE.}$$

$x \rightarrow 2^-$  means  $x$  approaches 2 from the left, i.e.  $x < 2$ . 

$x$  approaches 2 as 1, 1.9, 1.99, 1.999, .....

Therefore,  $x - 2 < 0$  (negative). and  $\lim_{x \rightarrow 2^-} \frac{1}{x - 2} = -\infty$

(★ NOT REQUIRED. DOES NOT COUNT IN YOUR QUIZ CREDIT. The problem is of actual exam difficulty level. It is recommended to try it to check whether you handle the materials well enough for the exam.)

Compute the limit (a finite number,  $+\infty$  or  $-\infty$ ):

$$\lim_{x \rightarrow -2^-} \frac{1-x}{x^2(x+2)}$$

Similar to 2(b), we have  $\frac{1-(-2)}{(-2)^2 \cdot (-2+2)} = \frac{+3}{(+4) \cdot (0)}$ ,

which is  $\infty$ . We need to determine whether it is  $+\infty$  or  $-\infty$ .

$x \rightarrow -2^-$  means  $x$  approaches  $-2$  from the left,



Therefore,  $x+2$  is negative as  $x \rightarrow -2^-$   
(e.g.  $-3+2 = -1$ )

$$1-x = 1-(-2) = 3 \text{ positive.}$$

$$x^2 = (-2)^2 = 4 \text{ positive.}$$

All together, we have

$$\lim_{x \rightarrow -2^-} \frac{1-x}{x^2(x+2)} = \boxed{-\infty}$$