Name: ______________________

1. (2 points) Complete the following indefinite integral formulas (from the lec-notes of sec4.4):

- $\int x^n dx = \frac{1}{n+1} \cdot x^{n+1} + C$, $n \neq -1$; $\int \sin x dx = -\cos x + C$; $\int \cos x dx = \sin x + C$

- $\int k dx = kx + C$, k is a constant; $\int \sec^2 x dx = \tan x + C$; $\int \sec x \tan x dx = \sec x + C$

2. (4 points) • $\frac{3+4\sqrt{t}}{\sqrt{t}}$ can be broken down into two parts as $\frac{3+4\sqrt{t}}{\sqrt{t}} = \frac{3}{\sqrt{t}} + 4$.
Use this to evaluate the integral

$$\int \frac{3+4\sqrt{t}}{\sqrt{t}}\, dt$$

$$= \int 3t^{-\frac{1}{2}} + 4\, dt = 3 \cdot \frac{1}{-\frac{1}{2}+1} \cdot t^{-\frac{1}{2}+1} + 4t + C$$

$$= \boxed{3 \cdot 2 \cdot \sqrt{t} + 4t + C}$$

- Expand the brackets $\sec x \cdot (2\sec x - \tan x) = 2\sec^2 x - \sec x \cdot \tan x$ and use this to evaluate $\int \sec x \cdot (2\sec x - \tan x)\, dx$

$$= \int 2\sec^2 x - \sec x \cdot \tan x \cdot dx$$

$$= \boxed{2\tan x - \sec x + C}$$

3. (4 points) • (FTC2) If $F'(x) = f(x)$, then $\int_a^b f(x)dx = F(x)\Big|_a^b = F(b) - F(a)$

- The average of $f(x)$ over $x \in [a, b]$ is defined as

$$f_{ave} = \frac{1}{b-a} \int_a^b f(x)\, dx.$$

- Compute the average of $f(x) = \cos x$ over $[0, \frac{\pi}{2}]$.

$$f_{ave} = \frac{1}{\frac{\pi}{2} - 0} \cdot \int_0^{\frac{\pi}{2}} \cos x\, dx = \frac{2}{\pi} \cdot \sin x \Big|_0^{\frac{\pi}{2}} = \frac{2}{\pi}\left(\sin\frac{\pi}{2} - \sin 0\right) = \boxed{\frac{2}{\pi}}$$

4. (4 points) To evaluate the integral $\int \sin(\sqrt{x}) \cdot \frac{1}{\sqrt{x}}\, dx$, we need the following steps:

- If $u = \sqrt{x}$, then $du = \underline{\frac{1}{2} \cdot x^{-\frac{1}{2}}}\, dx$. This implies $\frac{1}{\sqrt{x}}\, dx = \underline{\ 2\ }\, du$.
- Substituting $\sqrt{x}$ by u and $\frac{1}{\sqrt{x}}\, dx$ by $2du$, the integral of x is converted into

$$\int \sin(\sqrt{x}) \cdot \frac{1}{\sqrt{x}}\, dx = \int \underline{\sin(u) \cdot 2 \cdot}\, du$$

- Use the formula in Problem 1 to evaluate the integral $\int 2\sin u\, du = \underline{-2\cos u + C}$
- Use $u = \sqrt{x}$ again to change the above result of u back to x, we have

$$\int \sin(\sqrt{x}) \cdot \frac{1}{\sqrt{x}}\, dx = \int 2\sin u\, du = -2\cos u + C = \underline{-2\cos\sqrt{x} + C}$$

- In general, if $u = g(x)$, then $du = \underline{g'(x)}\, dx$. By substituting u, du, we have

$$\text{(U-sub formula)}: \quad \int f\big(g(x)\big) \cdot g'(x)dx = \underline{\int f(u) \cdot du}$$

5. (3 points) Linear U-sub:

- If $u = x + 13$, then $du = \underline{dx}$. By using this u-sub, we have

$$\int (x+13)^{99}\, dx = \int \underline{u^{99}}\, du = \underline{\frac{1}{100} \cdot u^{100} + C}\text{(a function of u)} = \underline{\frac{1}{100}(x+13)^{100} + C}\text{(a function of x)}$$

- To evaluate $\int \sec^2(5x-1)dx$ by u-sub, we should choose $u = \underline{5x-1}$, then $du = \underline{5 \cdot dx} \Longrightarrow$ $dx = \underline{\frac{1}{5}}\, du$. And finally,

$$\int \sec^2(5x-1)dx = \underline{\int \sec^2 u \cdot \frac{1}{5}\, du} = \underline{\frac{1}{5} \cdot \tan u + C} = \underline{\frac{1}{5}\tan(5x-1) + C}$$

6. (3 points) U-sub for definite integral:

- Use Problem 5 to evaluate

$$\int_{-13}^{-12} (x+13)^{99}\, dx = \int_{u=0}^{u=1} u^{99}\, du = \frac{1}{100} \cdot u^{100}\Big|_0^1 = \frac{1}{100} \cdot 1^{100} - 0 = \boxed{\frac{1}{100}}$$

- Let $u = \sqrt{x}$. If $x = 0$, then $u = \sqrt{0} = \underline{0}$. If $x = \frac{\pi^2}{4} = (\frac{\pi}{2})^2$, then $u = \sqrt{(\frac{\pi}{2})^2} = \underline{\frac{\pi}{2}}$.

$$\begin{aligned} \int_0^{\frac{\pi^2}{4}} \sin(\sqrt{x}) \cdot \frac{1}{\sqrt{x}}\, dx &= \int_{0}^{\frac{\pi}{2}} 2\sin u\, du \\ &= -2\cos u\Big|_{0}^{\frac{\pi}{2}} = \Big[-2\cos(\tfrac{\pi}{2})\Big] - \Big[-2\cos(0)\Big] = \boxed{0+2} \end{aligned}$$