Name: ____________________

Section: ____________ **Recitation/Instructor**: ____________________

INSTRUCTIONS

- Fill in your name, etc. on this first page.
- Without fully opening the exam, check that you have pages 1 through 11.
- **Show all your work** on the standard response questions. Write your answers clearly! Include enough steps for the grader to be able to follow your work. Don't skip limits or equal signs, etc. Include words to clarify your reasoning.
- Do first all of the problems you know how to do immediately. Do not spend too much time on any particular problem. Return to difficult problems later.
- If you have any questions please raise your hand and a proctor will come to you.
- You will be given exactly 90 minutes for this exam.
- Remove and utilize the formula sheet provided to you at the end of this exam.
- This is a practice exam. The actual exam may differ significantly from this practice exam because there are many varieties of problems that can test each concept.

ACADEMIC HONESTY

- Do not open the exam booklet until you are instructed to do so.
- Do not seek or obtain any kind of help from anyone to answer questions on this exam. If you have questions, consult only the proctor(s).
- Books, notes, calculators, phones, or any other electronic devices are not allowed on the exam. Students should store them in their backpacks.
- No scratch paper is permitted. If you need more room use the back of a page.
- Anyone who violates these instructions will have committed an act of academic dishonesty. Penalties for academic dishonesty can be very severe. All cases of academic dishonesty will be reported immediately to the Dean of Undergraduate Studies and added to the student's academic record.

I have read and understand the above instructions and statements regarding academic honesty: ____________________

SIGNATURE

Standard Response Questions. Show all work to receive credit. Please $\boxed{\textbf{BOX}}$ your final answer.

1. Consider the curve $y = f(x) = \dfrac{x^2-3}{x-2}$.

$f'(x) = \dfrac{(x-1)(x-3)}{(x-2)^2}$ $f''(x) = \dfrac{2}{(x-2)^3}$

   (a) (5 points) Find the intervals on which $f(x)$ is increasing and the intervals on which $f(x)$ is decreasing.

f' +++ | −−− o −−− | +++ at 1, 2, 3.

Domain : $(-\infty, 2) \cup (2, +\infty)$

$f' = 0 \Rightarrow (x-1)\cdot(x-3) = 0 \Rightarrow x=1, x=3$

Increasing : $(-\infty, 1] \cup [3, +\infty)$

Decreasing : $[1, 2) \cup (2, 3]$

   (b) (2 points) Find any local extreme values and identify whether each is a local minimum or local maximum.

$x=1$ Increasing $\to$ Decreasing, local max ; $x=3$, Decreasing $\to$ Increasing. local min.

   (c) (5 points) Find the intervals on which $f(x)$ is concave up and the intervals on which $f(x)$ is concave down.

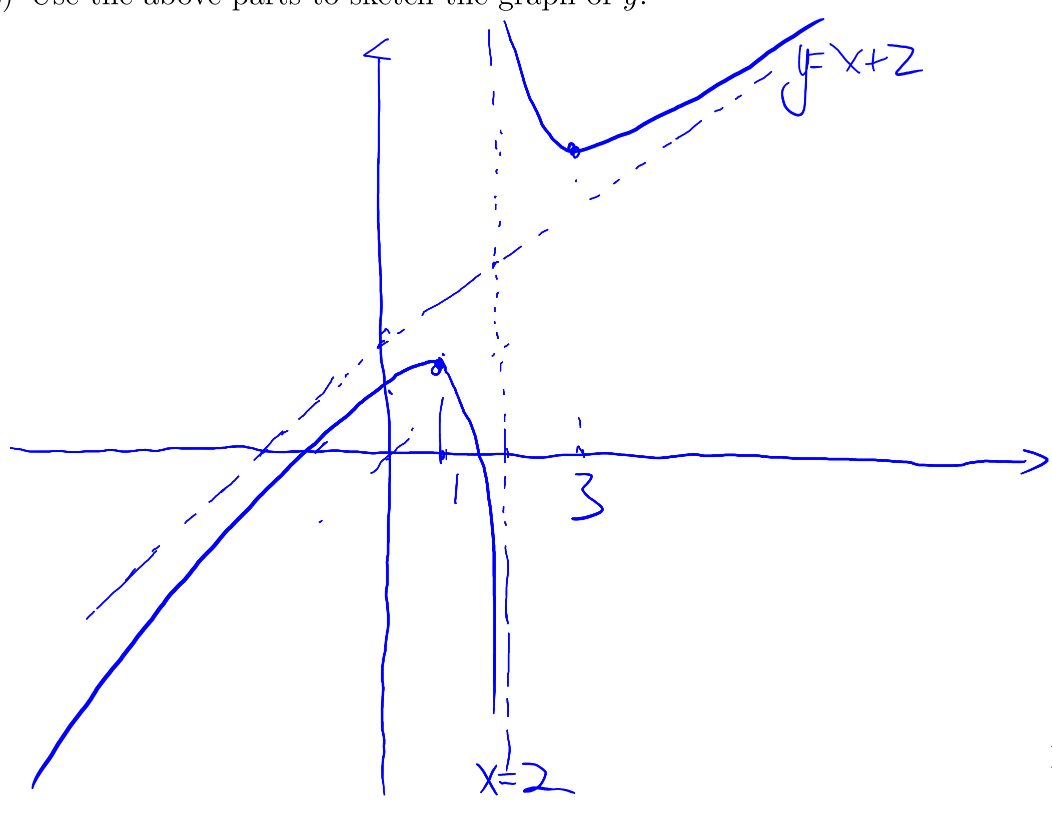
$x<2$ $f'' = \dfrac{2}{(x-2)^3} < 0$. Concave down $(-\infty, 2)$

$x>2$ $f'' = \dfrac{2}{(x-2)^3} > 0$ Concave up $(2, +\infty)$

   (d) (4 points) Give equations for all asymptotes.

Vertical asymptote: $x=2$. $f(x) = x+2+\dfrac{1}{x-2}$. Slant asymptote : $y = x+2$. H.A.: None.

   (e) (2 points) Use the above parts to sketch the graph of y.



2. Find the most general antiderivative of the following functions

(a) (6 points) $f(x) = \dfrac{3}{x^3} + 5x$

$f(x) = 3 \cdot x^{-3} + 5x$

Anti-D: $F(x) = 3 \cdot \frac{1}{-3+1} \cdot x^{-3+1} + 5 \cdot \frac{1}{2} x^2$

$= -\frac{3}{2} \cdot x^{-2} + \frac{5}{2} \cdot x^2 + C$

(b) (6 points) $g(x) = \cos^2(x/2)$ **Hint:** *use the double angle formula for cosine:* $\cos(2\theta) = 2\cos^2(\theta) - 1$

$g(x) = \cos^2\left(\frac{x}{2}\right)$, $\theta = \frac{x}{2}$ $\Rightarrow \cos^2(\theta) = \frac{\cos(2\theta)+1}{2}$

$= \frac{\cos\left(2 \cdot \frac{x}{2}\right) + 1}{2} = \frac{\cos x + 1}{2} = \frac{1}{2} \cdot \cos x + \frac{1}{2}$

anti D: $G(x) = \frac{1}{2} \cdot \sin x + \frac{1}{2} \cdot x + C$

(c) (6 points) $h(t) = \dfrac{3t^3 + t^2}{2\sqrt{t}}$

$h(t) = \frac{3t^3}{2t^{\frac{1}{2}}} + \frac{t^2}{2 \cdot t^{\frac{1}{2}}} = \frac{3}{2} \cdot t^{3-\frac{1}{2}} + \frac{1}{2} \cdot t^{2-\frac{1}{2}}$

$= \frac{3}{2} \cdot t^{\frac{5}{2}} + \frac{1}{2} \cdot t^{\frac{3}{2}}$

Anti-D: $\frac{3}{2} \cdot \frac{1}{\frac{5}{2}+1} t^{\frac{5}{2}+1} + \frac{1}{2} \cdot \frac{1}{\frac{3}{2}+1} t^{\frac{3}{2}+1} + C$

$= \boxed{\frac{3}{2} \cdot \frac{2}{7} t^{\frac{7}{2}} + \frac{1}{2} \cdot \frac{2}{5} t^{\frac{5}{2}} + C}$

3. An object is moving along a straight vertical line. Given its acceleration, $a(t) = -2t + 2$, velocity at $t = 1$, $v(1) = 4$ and height at $t = 0$, $h(0) = 3$,

(a) (6 points) Find the object's velocity, $v(t)$, as a function of time.

$v(t)$ is the anti-D of $a(t) = -2t+2$

$v(t) = -2\cdot\frac{1}{2}t^2 + 2t + C = -t^2 + 2t + C$

$4 = v(1) = -1^2 + 2\cdot 1 + C = 1 + C \Rightarrow 4 = 1 + C \Rightarrow C = 3$

$v(t) = -t^2 + 2t + 3$

(b) (6 points) Find the object's height, $h(t)$, as a function of time.

h is the anti-D of $v(t) = -t^2 + 2t + 3$

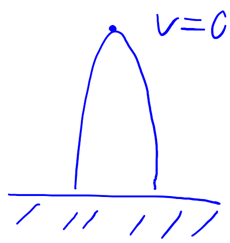
$h(t) = -\frac{1}{2+1}\cdot t^{2+1} + 2\cdot\frac{1}{2}t^2 + 3t + C$

$= -\frac{1}{3}t^3 + t^2 + 3t + C$

$3 = h(0) = 0 + 0 + 0 + C \Rightarrow C = 3. \Rightarrow h(t) = -\frac{1}{3}t^3 + t^2 + 3t + 3$

(c) (6 points) When is the object at its highest position? (Assume $t \geq 0$.)

where $v = 0$.



$v(t) = -t^2 + 2t + 3 = 0$

$(-t+3)(t+1) = 0$

$\Rightarrow -t+3 = 0, \quad t+1 = 0 \Rightarrow \boxed{t=3}$ or $\cancel{t=-1}$

4. Calculate the following derivatives.

(a) (5 points) $\frac{d}{dx}\int_x^0 \frac{t^2}{1+t^4}dt$ $= \frac{d}{dx}\left(-\int_a^x \frac{t^2}{1+t^4}dt\right)$

$= \boxed{-\frac{x^2}{1+x^4}}$

formula: $\left(\int_a^x f(t)dt\right)' = f(x)$

(b) (6 points) $\frac{d}{dx}\int_1^{\sqrt{x}} \frac{dt}{(1+t)^2}$

$= \frac{1}{(1+\sqrt{x})^2}(\sqrt{x})' = \boxed{\frac{1}{(1+\sqrt{x})^2}\cdot\frac{1}{2}\cdot x^{-\frac{1}{2}}}$

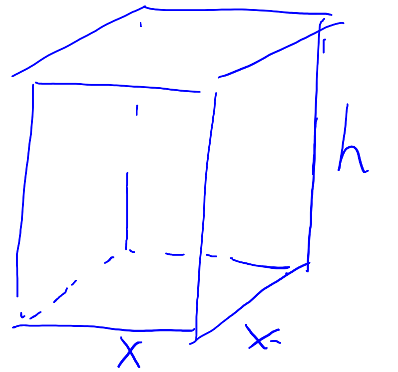
(c) (7 points) $\frac{d}{dx}\int_x^{x^2} \frac{\sin^2(t)}{t}\, dt$

$= \frac{\sin^2(x^2)}{x^2}(x^2)' - \frac{\sin^2(x)}{x}x'$

$= \boxed{\frac{\sin^2(x^2)}{x^2}\cdot 2x - \frac{\sin^2(x)}{x}\cdot 1}$

5. (18 points) Suppose you want to build a fish tank in the shape of a right rectangular box with square base and no top which will hold 6 cubic feet of water. The glass for the sides costs \$1 per square foot, and the metal for the bottom costs \$1.50 per square foot. What dimensions for the tank will minimize the cost?

$V = 6 = x^2 \cdot h \Rightarrow h = \frac{6}{x^2}$



$\text{Cost} = 4x \cdot h + 1.5x^2$

$f(x) = 4x \cdot \frac{6}{x^2} + 1.5x^2$

$= \frac{24}{x} + 1.5 \cdot x^2$ Domain: $x > 0$ $(0, +\infty)$

$f'(x) = -\frac{24}{x^2} + 1.5 \cdot 2x = -\frac{24}{x^2} + 3x = \frac{-24 + 3x^3}{x^2}$

$= \frac{3(x^3 - 8)}{x^2}$

$f'(x) = 0 \Rightarrow x^3 - 8 = 0 \Rightarrow x = 2$.

$0 < x < 2$, $x^3 < 8$, $f'(x) < 0$ f': $- - -$ $+ + +$

$x > 2$. $x^3 > 8$, $f'(x) > 0$ 0 2

f attains min at $x = 2$.

The min cost Dimension: base $\boxed{x = 2}$

$= f(2) = \frac{24}{2^2} + 1.5 \cdot 2^2$ height $\boxed{h = \frac{6}{x^2} = \frac{6}{4}.}$

Multiple Choice. Circle the best answer. No work needed. No partial credit available.

6. (7 points) Write the following sum in Σ-notation: $6+12+24+48+96+192$

A. $\sum_{i=1}^{6} 6i$ $= 6+12+18+24+\cdots+36$

B. $\sum_{i=0}^{5} 6+6i$ $=(6+0)+(6+6)+(6+12)+\cdots+(6+30)=6+12+\cdots+36$

C. $\sum_{i=0}^{5} 6(2^i)$ $=6\cdot 1+6\cdot 2+6\cdot 2^2+6\cdot 2^3+\cdots+6\cdot 2^5=6+12+24+48+\cdots+6\cdot 32$ $\underset{192}{\shortparallel}$

D. $\sum_{i=1}^{6} 6(2^i)$ $=6\cdot 2+6\cdot 2^2+\cdots+6\cdot 2^6=12+24+\cdots+\underset{384}{\underset{\shortparallel}{6\cdot 64}}$

E. None of the above.

7. (7 points) Write the following sum in Σ-notation: $7-11+15-19+23-27+31-35+39$

A. $\sum_{i=1}^{9}(-1)^i(4i+3)$ $=-(4+3)+1\cdot(8+3)-\cdots-$

B. $\sum_{i=0}^{8}(-1)^{i+1}(4i+7)$ $=(-1)^{0+1}(7)+(-1)^2(4+7)\cdots$

C. $\sum_{i=0}^{8}(-1)^i(4i+3)$ $=(-1)^0(0+3)+(-1)^1(4+3)\cdots$

D. $\sum_{i=1}^{9}(-1)^{i+1}(4i+3)$ $=(-1)^{1+1}(4+3)+(-1)^{2+1}(8+3)+(-1)^{3+1}(12+3)+\cdots$

E. None of the above. $=7-11+15-\cdots$

8. (7 points) Find the numerical value of the sum: $\sum_{i=1}^{2016}(-1)^i i$

A. -2016

B. -1008

C. 0

D. 1008

E. 2016

$=(-1+2)(-3+4)(-5+6)(-7+8)\cdots(-2015+2016)$

$= \underbrace{1+1+1\cdots+1}_{\frac{2016}{2}=1008 \text{ pairs}}$

$=1008.$

9. (7 points) Evaluate the definite integral: $\int_1^4 (t^2+1)\sqrt{t}\,dt$

A. $\frac{2}{7}(4^{7/2}-1)+\frac{2}{3}(4^{3/2}-1)$

B. $\frac{2}{5}(4^{5/2}-1)+2(4^{1/2}-1)$

C. $\frac{2}{7}(4^{7/2})+\frac{2}{3}(4^{3/2})$

D. $\frac{2}{5}(4^{5/2})+2(4^{1/2})$

E. None of the above.

$f(t)=(t^2+1)\sqrt{t}=t^2\cdot t^{\frac{1}{2}}+t^{\frac{1}{2}}=t^{\frac{5}{2}}+t^{\frac{1}{2}}$

Anti-D: $F(t)=\frac{1}{\frac{5}{2}+1}\cdot t^{\frac{5}{2}+1}+\frac{1}{\frac{1}{2}+1}\cdot t^{\frac{1}{2}+1}=\frac{2}{7}t^{\frac{7}{2}}+\frac{2}{3}t^{\frac{3}{2}}$

$\int_1^4 (t^2+1)\sqrt{t}\,dt=F(t)\Big|_1^4=\left(\frac{2}{7}4^{\frac{7}{2}}+\frac{2}{3}\cdot 4^{\frac{3}{2}}\right)-\left(\frac{2}{7}+\frac{2}{3}\right)$

$=\frac{2}{7}\cdot 4^{\frac{7}{2}}+\frac{2}{3}4^{\frac{3}{2}}-\frac{2}{7}-\frac{2}{3}$

10. (7 points) Evaluate the definite integral: $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{4}{\cos^2(z)}\,dz$

$\sec^2(z)=\frac{1}{\cos^2(z)}$

A. $4\frac{1}{\sqrt{3}}-4\sqrt{3}$

B. $4\sqrt{3}-4\frac{1}{\sqrt{3}}$

C. $4\frac{1}{\sqrt{2}}-4\sqrt{2}$

D. $4\sqrt{2}-4\frac{1}{\sqrt{2}}$

E. None of the above.

$=\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} 4\sec^2(z)\,dz$

$=4\cdot\tan(z)\Big|_{\frac{\pi}{6}}^{\frac{\pi}{3}}$

$=4\tan(\frac{\pi}{3})-4\tan(\frac{\pi}{6})$

$=4\sqrt{3}-4\cdot\frac{1}{\sqrt{3}}$

11. (7 points) Evaluate the definite integral: $\int_0^{\frac{\pi}{4}} 3\cos(2x)\,dx$

A. $\frac{1}{2}$

B. $\frac{3}{2}$

C. $\frac{3\sqrt{2}}{4}$

D. $\frac{\sqrt{2}}{4}$

E. None of the above.

$\sin(2x)' = 2\cdot\cos(2x)$

anti-D of $\cos(2x)$ is $\frac{1}{2}\sin(2x)$

$\int_0^{\frac{\pi}{4}} 3\cdot\cos(2x)\,dx=3\cdot\frac{1}{2}\sin(2x)\Big|_0^{\frac{\pi}{4}}$

$=\frac{3}{2}\sin(2\cdot\frac{\pi}{4})-\frac{3}{2}\cdot\sin 0$

$=\frac{3}{2}\sin\frac{\pi}{2}-0$

$=\frac{3}{2}$

12. (7 points) Consider the function $f(x) = \sqrt[3]{x}$. The linearization of f at $x = 1$ is $L(x) =$

A. $\frac{x-1}{3}$

B. $1 + \frac{x}{3}$

C. $\frac{x+2}{3}$

D. $1 - \frac{x-1}{3}$

E. None of the above.

$= x^{\frac{1}{3}}$ $a=1$.

$f'(x) = \frac{1}{3} \cdot x^{\frac{1}{3}-1} = \frac{1}{3} \cdot x^{-\frac{2}{3}}$.

$a=1$ $f(1) = \sqrt[3]{1} = 1$, $f'(1) = \frac{1}{3} \cdot 1^{-\frac{2}{3}} = \frac{1}{3}$.

$L(x) = 1 + \frac{1}{3} \cdot (x-1) = \frac{1}{3}x + \frac{2}{3}$.

13. (7 points) Estimate $\sqrt[3]{1.1}$ using linearization of $\sqrt[3]{x}$.

A. $\frac{29}{30}$

B. $\frac{30}{30}$

C. $\frac{31}{30}$

D. $\frac{32}{30}$

E. $\frac{33}{30}$

$x = 1.1$.

$L(1.1) = \frac{1}{3} \cdot 1.1 + \frac{2}{3} = \frac{11}{30} + \frac{20}{30}$

$= \frac{31}{30}$.

14. (7 points) Use Newton's method to approximate the positive root of the function $x^2 - 3$. Starting with an initial guess of $x_1 = 2$, find x_2.

A. $x_2 = \frac{7}{4}$

B. $x_2 = -\frac{1}{4}$

C. $x_2 = 1$

D. $x_2 = \frac{9}{4}$

E. None of the above.

$f(x) = x^2 - 3$ $f(2) = 2^2 - 3 = 1$

$f'(x) = 2x$ $f'(2) = 4$

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

$= 2 - \frac{1}{4}$

Congratulations you are now done with the exam!
Go back and check your solutions for accuracy and clarity. Make sure your final answers are $\boxed{\text{BOXED}}$.

When you are completely happy with your work please bring your exam to the front to be handed in. **Please have your MSU student ID ready** so that is can be checked.

DO NOT WRITE BELOW THIS LINE.

Page	Points	Score
2	18	
3	18	
4	18	
5	18	
6	18	
7	21	
8	21	
9	21	
Total:	153	

No more than 150 points may be earned on the exam.

FORMULA SHEET

Algebraic

- $a^2 - b^2 = (a-b)(a+b)$
- $a^3 - b^3 = (a-b)(a^2+ab+b^2)$
- Quadratic Formula: $\dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

Geometric

- Area of Circle: πr^2
- Circumference of Circle: $2\pi r$
- Circle with center (h,k) and radius r:

$$(x-h)^2 + (y-k)^2 = r^2$$

- Distance from (x_1, y_1) to (x_2, y_2):

$$\sqrt{(x_1-x_2)^2 + (y_1-y_2)^2}$$

- Area of Triangle: $\frac{1}{2}bh$
- $\sin\theta = \dfrac{\text{opposite leg}}{\text{hypotenuse}}$
- $\cos\theta = \dfrac{\text{adjacent leg}}{\text{hypotenuse}}$
- $\tan\theta = \dfrac{\text{opposite leg}}{\text{adjacent leg}}$
- If $\triangle ABC$ is similar to $\triangle DEF$ then

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$$

- Volume of Sphere: $\frac{4}{3}\pi r^3$
- Surface Area of Sphere: $4\pi r^2$
- Volume of Cylinder/Prism: (height)(area of base)
- Volume of Cone/Pyramid: $\frac{1}{3}$(height)(area of base)

Trigonometric

- $\sin^2\theta + \cos^2\theta = 1$
- $\sin(2\theta) = 2\sin\theta\cos\theta$
- $\cos(2\theta) = \cos^2\theta - \sin^2\theta = 1 - 2\sin^2\theta = 2\cos^2\theta - 1$

Limits

- $\lim\limits_{x\to a} f(x) = L$ if for every $\varepsilon > 0$ there exists $\delta > 0$ so that $|f(x) - L| < \varepsilon$ when $|x-a| < \delta$.
- $\lim\limits_{x\to a} f(x)$ exists if and only if

$$\lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x)$$

- $\lim\limits_{\theta\to 0} \dfrac{\sin\theta}{\theta} = 1$
- $\lim\limits_{\theta\to 0} \dfrac{1-\cos\theta}{\theta} = 0$

Derivatives

- $f'(x) = \lim\limits_{h\to 0} \dfrac{f(x+h) - f(x)}{h}$
- $(fg)' = f'g + fg'$
- $\left(\dfrac{f}{g}\right)' = \dfrac{f'g - fg'}{g^2}$
- $(f(g(x)))' = f'(g(x)) \cdot g'(x)$
- $(\sin x)' = \cos x$
- $(\cos x)' = -\sin x$
- $(\tan x)' = \sec^2 x$
- $(\sec x)' = \sec x \cdot \tan x$

Theorems

- (IVT) If f is continuous on $[a,b]$, $f(a) \neq f(b)$, and N is between $f(a)$ and $f(b)$ then there exists $c \in (a,b)$ that satisfies $f(c) = N$.
- (MVT) If f is continuous on $[a,b]$ and differentiable on (a,b) then there exists $c \in (a,b)$ that satisfies $f'(c) = \dfrac{f(b)-f(a)}{b-a}$.
- (FToC P1) If $F(x) = \displaystyle\int_a^x f(t)\,dt$ then $F'(x) = f(x)$.

Other Formulas

- Newton's Method: $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$
- Linearization of f at a: $L(x) = f(a) + f'(a)(x-a)$
- $\displaystyle\sum_{i=1}^{n} c = cn$
- $\displaystyle\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$
- $\displaystyle\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$