Q1[*Sec1.4, Average rate of change/Average velocity, see also Q9*] Let $f(x) = \cos x + 2$. Compute the average rate of change of $f(x)$ on the interval $[0, \frac{\pi}{2}]$

Average rate of change of $f(x)$ over $[a,b]$

$= \dfrac{f(b)-f(a)}{b-a}$

Q2[*Sec1.5/1.6, Limit and Limit Laws*] Evaluate the following limits

(a)Direct plug in-type

Suppose $\lim\limits_{x\to 4} f(x) = 2, \lim\limits_{x\to 4} g(x) = 3.$ Find $\lim\limits_{x\to 4} \dfrac{xf(x)+2}{f(x)-\sqrt{g(x)}}$

Direct Plug in 4

and use linear properties of limit.

(b)$\frac{1}{0}$-type/One-sided limits

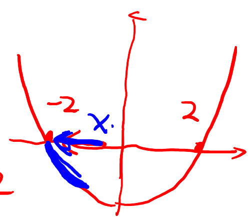
$$\lim_{x\to 0^+} \frac{x-3}{x^2(x+5)}$$

Plug in 0, we have $\frac{0-3}{0\cdot 5} \sim \frac{1}{0}$-type.

figure out the sign

☆☆**(c)Cancellation-type**

$$\lim_{x\to -2^+} \frac{|x^2-4|}{x+2}$$

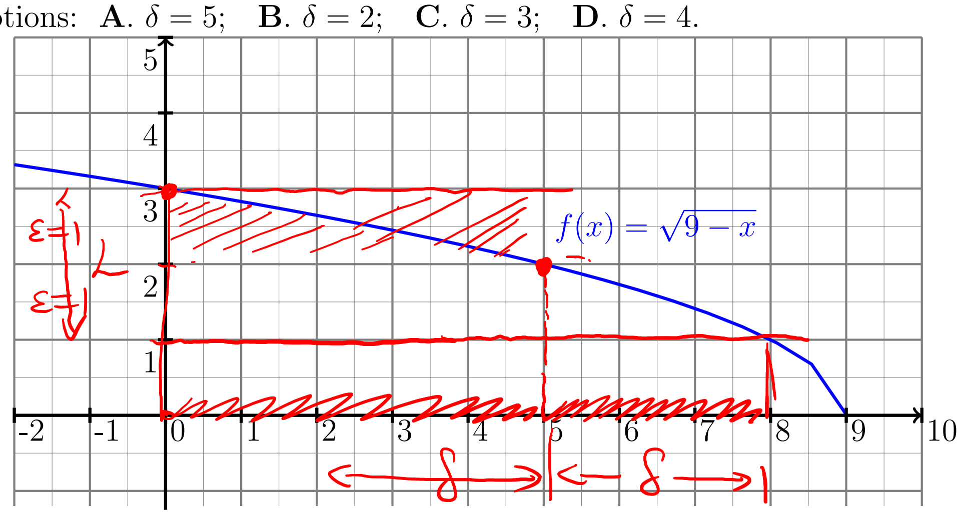


when $x \to -2^+$, $x > -2$

$\Rightarrow x^2-4<0$ or $4-x^2>0$

$\Rightarrow |x^2-4| = 4-x^2 = (2+x)(2-x)$

☆☆**Q3**[*Sec1.7, Limit Definition*] For $f(x) = \sqrt{9-x}; L = 2, a = 5, \varepsilon = 1$, use the graph of $f(x)$ to find the largest value of δ of $|x-a| < \delta$ in the formal definition of a limit which ensures that $|f(x) - L| < \varepsilon$.
Options: **A**. $\delta = 5$; **B**. $\delta = 2$; **C**. $\delta = 3$; **D**. $\delta = 4$.



Q4[*Sec1.8, Domain of continuity*] Use interval notation to indicate where $f(x)$ is continuous.

(a)

$$f(x) = \frac{x^2 - 3x + 1}{x - 3}.$$ Choose from below The x for which the denominator is not zero.

A. $(-\infty, +\infty)$; **B**. $(-\infty, 3) \cup (3, +\infty)$; **C**. $(-\infty, 1) \cup (1, +\infty)$; **D**. $(-\infty, 1) \cup (1, 3) \cup (3, +\infty)$.

(b)

$f(x) = \sqrt{x+1}$. Choose from below The x for which below root is positive.

A. $(-\infty, +\infty)$; **B**. $(-\infty, -1)$; **C**. $[-1, +\infty)$; **D**. $(1, +\infty)$.

(c)

$$f(x) = \frac{(x^2 - 3x + 1)\sqrt{x+1}}{x - 3}.$$ Use (a,b) to indicate the intervals of continuous for (c)

Q5[*Sec1.8, Piecewise function*] For what value of k will $f(x)$ be continuous for all values of x ?

$$f(x) = \begin{cases} \frac{x^2 - 3k}{x - 3}, & x \le 2 \\ 8x - k, & x > 2 \end{cases}$$

Plug x=2 into the fist and second formulas, then set them equal.

A $k = 2$;

B $k = 3$;

C $k = 4$;

D $k = 5$.

Q6[*Sec1.8, Intermediate Value Theorem(IVT)*] Suppose function $h(x)$ is continuous on $[0, 4]$. Suppose $h(0) = 2, h(1) = 0, h(2) = -3, h(3) = 2, h(4) = 5$. For what value of N, the must be a $c \in (3, 4)$ such that $h(c) = N$?

Draw the picture of h(x)
and find the intermediate
value N in (3,4)

A $N = 0.5$

B $N = 0$

C $N = -2$

D $N = 2.5$

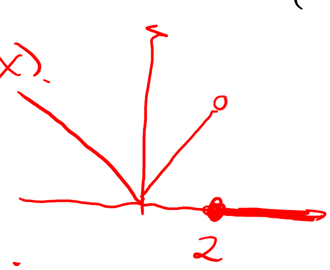
Q6[*Sec2.1/2.2, derivative at given point*] Select all true statements about the function $f(x) = \begin{cases} |x|, & x < 2 \\ 0, & x \ge 2 \end{cases}$

I $f(x)$ is differentiable at $x = 0$

II $f(x)$ is continuous at $x = 2$

III $\lim_{x \to 0} f(x)$ exists

Draw the picture of f(x).
Find the jumping points
and sharp-turning points.



Q7[*Sec2.1/2.2, definition of derivative*] Let $f(x) = \frac{1}{x+1}$

(a) ***[Derivative as a limit]*** Use the definition of the derivative to find $f'(x)$. (Your calculation must include computing a limit.)

Definition of derivative.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

(b) ***[Evaluating the derivative function at given point]*** Find $f'(2)$

Plug 2 into $f'(x)$ you find in (a)

(c) ***[Point-slope formula for the tangent line]*** Use part (b) to find an equation of a tangent line of $f(x)$ at $x = 2$.

Point-slope formula for tangent line at $x=a$

$$y = f'(a) \cdot (x-a) + f(2)$$

☆ **Q7***[*Sec2.1/2.2, definition of derivative*] Use the definition of the derivative to find $g'(1)$ for $g(x) = 2\sqrt{x}$.

To simplify $\frac{2\sqrt{x+h} - 2\sqrt{x}}{h}$

use the conjugation trick:

$$\sqrt{A} - \sqrt{B} = \frac{(\sqrt{A} - \sqrt{B})(\sqrt{A} + \sqrt{B})}{\sqrt{A} + \sqrt{B}}$$

$$= \frac{A - B}{\sqrt{A} + \sqrt{B}}.$$

Q8[*Sec2.3/2.4/2.5, Differentiation Formulas/Laws*] Find the derivatives of the following functions. Do not need to simplify.

(a)[Linear Rule+Power functions]

$$T(x) = 2\sqrt{x} - \frac{1}{2\sqrt{x}}$$

$\sqrt{x} = x^{\frac{1}{2}}$, $\frac{1}{x^a} = x^{-a}$,

$(x^n)' = n \cdot x^{n-1}$

(b)[Product Rule+Power functions]

$$g(t) = \underbrace{\Big(\frac{1}{t^5} - 2t\Big)}_{f}\underbrace{\Big(\frac{1}{\sqrt{t}} + \pi\Big)}_{g}.$$

product rule

(c)[Trig functions+Chain Rule]

$$y = \sin(x^2)$$

Outer: $\sin \blacksquare$

Inner: x^2

(c*)[Trig functions+Chain Rule]

$$y = \sin^2(x) = [\sin x]^2$$

Outer: $\boxed{}^2$

Inner: $\sin x$.

☆ **(d)[Quotient Rule+Trig functions+Chain Rule]**

$$f(t) = \frac{3t}{\tan(t^2-1)}$$

quotient rule.

then apply chain rule to

$[\tan(t^2-1)]'$

☆☆ **(e)[Trig functions+Double Chain Rule]**

$$f(x) = -2\sec\big(\cos(x^2+x)\big)$$

1st chain: outer: $-2\sec \blacksquare$

inner: $\cos(x^2+x)$

2nd chain $\rightarrow$ outer: $\cos \bullet$

inner: x^2+x.

Q9[*Sec2.7, Rates of Change/Functions of motion*] A particle moves according to the law of motion $s(t) = t^3 - 5t^2 + 6t$, where t is measured in seconds and s in feet

(a)[1.4, Average velocity] Find the average velocity over the interval $[0, 2]$.

$V_{ave} = \frac{S(t_2) - S(t_1)}{t_2 - t_1}$

(b)[Velocity and position] Find the velocity at time t.

$v(t) = S'(t)$

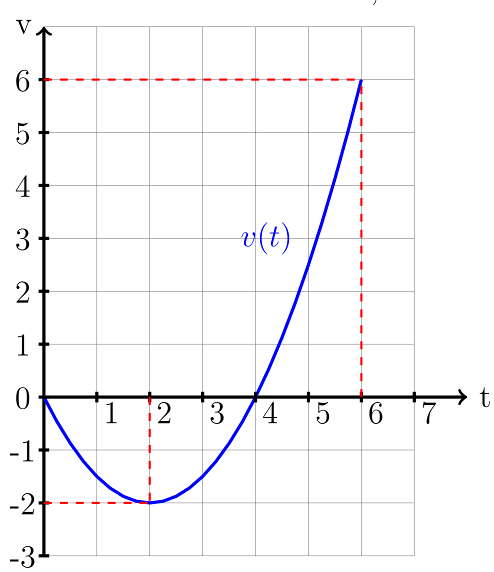
(c)[Acceleration and velocity] What is the acceleration after 6 seconds?

$a(t) = V'(t)$. Plug in 6 to get $a(t)$

(d)[Velocity and speed] What is the speed of the particle when the acceleration is zero?

speed $= |V(t)|$

Q10[*Sec2.7, Graph of the velocity*] The accompanying figure shows the velocity $v(t)$ of a particle moving on a horizontal coordinate line, for t in the closed interval $[0, 6]$.



(a) When does the particle move forward?

move forward: $v > 0$

(b) When does the particle slow down?

slow down: speed drops: $|v|$ is decreasing.

(c) When is the particle's acceleration positive?

$a(t) = V'(t) > 0 \Leftrightarrow V(t)$ is increasing

(d) When does the particle move at its greatest speed in $[0, 6]$?

highest (or lowest) point in the graph

Q11[*Sec2.6, Implicit differentiation*] Consider the curve $y^2 + 2xy + x^3 = x$

(a) Find the slope of the tangent line of the curve at the point $(1, -2)$.

(b) Find the equation of the tangent line of the curve at the point $(1, -2)$.

(a). Take derivative (w.r.t. x) both sides of the equation.

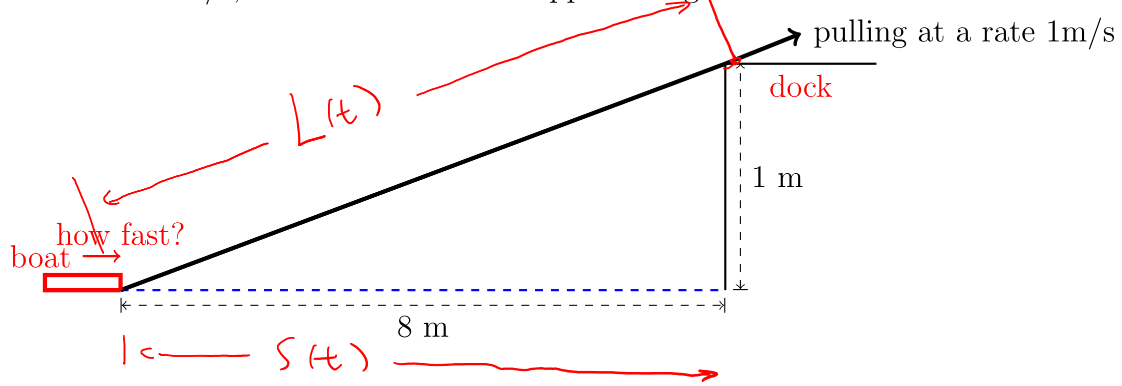
Consider $y = y(x)$ as an implicit function.

Plug in $x=1$, $y=-2$. and solve for y' (as a number)

slope $= y'$ at $x=1$.

(b) Point-slope formula for $(1,-2)$

☆☆ **Q12, *Sec2.8, Related Rates*** A boat is pulled into a dock by a rope attached to the bow of the boat and passing through a pulley on the dock that is 1 m higher than the bow of the boat. If the rope is pulled in at a rate of 1 m/s, how fast is the boat approaching the dock when it is 8 m from the dock?



Target functions: $S(t)$. $L(t)$.

Want to find $S'(t)$ when $S=8$

We know $L'(t) = 1$

$S=8 \Rightarrow L = \sqrt{64+1} = \sqrt{65}$.

Relation between S and L (from the graph)

$$S^2 + 1^2 = L^2 \qquad S^2(t) + 1 = L^2(t)$$

Take derivative w.r.t. t.

Plug in $S=8$, $L=\sqrt{65}$, $L'=1$ to solve for $S'(t)$