Formulas for 132 Final Exam

1.5/1.6 $\lim_{x\to a} f(x)$ limit by plug in $x = a$; $\lim_{x\to 0^+} \frac{1}{x} = +\infty$; $\lim_{x\to 0^-} \frac{1}{x} = -\infty$;

$\frac{0}{0}$-type limit cancelation: $\lim_{x\to a} \frac{x^2-a^2}{x-a} = \lim_{x\to a} x + a = 2a$.

1.8 (IVT) $f(a) > N, f(b) < N$ or $f(a) < N, f(b) > N \Longrightarrow f(c) = N$ for some $c \in (a,b)$

2.1/2.2 $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$. Tangent line of f at $x = a$: $y = f(a) + f'(a)(x-a)$

2.3 $(x^n)' = nx^{n-1}; (constant)' = 0; (kx+m)' = k$

$(f\cdot g)' = f'\cdot g + f\cdot g'$; $(\frac{f}{g})' = \frac{f'\cdot g - f\cdot g'}{g^2}$

2.4 $(\sin x)' = \cos x; (\cos x)' = -\sin x; (\tan x)' = \sec^2 x; (\sec x)' = \sec x \cdot \tan x$;

$\lim_{\theta\to 0} \frac{\sin\theta}{\theta} = 1$; $\lim_{\blacksquare\to 0} \frac{\sin\blacksquare}{\blacksquare} = 1$

2.5 $(f(g))' = f'(g)\cdot g'$; Triple chain rule: $(f(g(h)))' = f'(g(h))\cdot g'(h)\cdot h'$

2.6 Implicit differential rule: $y = y(x)$, $\frac{d}{dx}y = y'$; $\frac{d}{dx}y^2 = 2y\cdot y'$; $\frac{d}{dx}y^3 = 3y^2\cdot y'$; $\frac{d}{dx}(xy) = y + x\cdot y'$; $\cdots$

Tangent line at $x = a, y = b, \frac{dy}{dx} = k$: $y = b + k(x-a)$

2.7 Graph of velocity $v(t)$. $s'(t) = v(t); v'(t) = a(t)$. Speed up: $|v|$ is increasing.

2.8 Related rates: chain rule+implict diff rule.

2.9 Linearization. $L(x) = f(a) + f'(a)(x-a) \approx f(x)$

3.2 MVT. $f'(c) = \frac{f(b)-f(a)}{b-a}$

3.3 Increasing: $f'(x) > 0$; Decreasing: $f'(x) < 0$; Concave up: $f''(x) > 0$; Concave down: $f''(x) < 0$

Critical points: $f'(x) = 0$; Inflection points: $f''(x) = 0$

3.4 $\frac{1}{0^\pm} = \pm\infty$; $\frac{1}{\infty} = 0$. Highest term rule.

3.7 Optimization. Find target function$f(x) \longrightarrow$ solve $f'(x) = 0 \longrightarrow$ show that is max/min.

3.8 Newton's Method: $x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$ approximating x for $f(x) = 0$

4.2 Find $\int_a^b f(x)dx, \int_a^b |f(x)|dx$ from the graph. $\int_a^a f(x)dx = 0$; $\int_a^b 1dx = b-a$; $\int_a^b Cdx = C(b-a)$

$\int_a^b f(x)dx = -\int_b^a f(x)dx$; $\int_a^c f(x)dx = \int_a^b f(x)dx + \int_b^c f(x)dx$;

4.3 $\left(\int_a^{u(x)} f(t)\ dt\right)' = f\big(u(x)\big)\cdot u'(x)$; $\int_a^b f(x)\ dx = F(x)|_a^b = F(b) - F(a)$

4.4 $\int x^n dx = \frac{1}{n+1}x^{n+1} + C$; $\int kdx = kx + C$; $\int \sin x dx = -\cos x + C$; $\int \cos x dx = \sin x + C$;

$\int \sec^2 x dx = \tan x + C$; $\int \sec x \tan x dx = \sec x + C$;

$\sin 0 = 0, \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}, \sin\frac{\pi}{2} = 1, \sin\pi = 0$; $\cos 0 = 1, \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}, \cos\frac{\pi}{2} = 0, \cos\pi = -1$; $\tan 0 = 0, \tan\frac{\pi}{4} = 1$

4.5 $u = g(x) \Longrightarrow du = g'(x)dx$; $\int f\big(g(x)\big)\cdot g'(x)dx = \int f(u)du$.

If $f(-x) = -f(x)$ (f is odd), then $\int_{-a}^a f(x)dx = 0$.

5.1 Top: $y = f(x)$, Bot: $y = g(x)$; Intersections: solve $f(x) = g(x) \Longrightarrow x = a, x = b$; Area= $\int_a^b f(x) - g(x)\ dx$