Practice Mid 1, Sec39. Fall 2018

Q1[*Sec1.4, Average rate of change/Average velocity, see also Q9*] Let $f(x) = \cos x + 2$. Compute the average rate of change of $f(x)$ on the interval $[0, \frac{\pi}{2}]$?

Solution: Average rate of change:

$$A.R.o.C. = \frac{f(\frac{\pi}{2}) - f(0)}{\frac{\pi}{2} - 0} = \frac{(\cos(\frac{\pi}{2}) + 2) - (\cos 0 + 2)}{\frac{\pi}{2} - 0} \quad (1)$$

$$= \frac{(0+2) - (1+2)}{\frac{\pi}{2}} = \frac{-1}{\frac{\pi}{2}} = -\frac{2}{\pi} \quad (2)$$

□

Q2[*Sec1.5/1.6, Limit and Limit Laws*] Evaluate the following limits

(a)Direct plug in-type

$$\lim_{x \to 0} \sqrt{\frac{x^2}{\cos x + 2}}$$

Solution:

$$\lim_{x \to 0} \sqrt{\frac{x^2}{\cos x + 2}} = \sqrt{\frac{0^2}{\cos 0 + 2}} = \sqrt{\frac{0}{1+2}} = \sqrt{0} = 0$$

(b)$\frac{1}{0}$-type/One-sided limits

$$\lim_{x \to 0^+} \frac{x-3}{x(x+5)} \qquad \lim_{x \to 0^-} \frac{x-3}{x(x+5)} \qquad \lim_{x \to 0} \frac{x-3}{x(x+5)}$$

Solution:

$$\lim_{x \to 0^+} \frac{x-3}{x(x+5)} = \frac{0-3}{0^+(0+5)} = \frac{-3}{0^+(5)} = -\infty, \qquad \lim_{x \to 0^+} \frac{x-3}{x(x+5)} = \frac{0-3}{0^-(0+5)} = \frac{-3}{0^-(5)} = +\infty$$

$$\lim_{x \to 0} \frac{x-3}{x(x+5)} \qquad \text{D.N.E.}$$

(c)Absolute value

$$\lim_{x \to 0^-} \frac{x}{|x|} \qquad \lim_{x \to 0^+} \frac{x}{|x|} \qquad \lim_{x \to 0} \frac{x}{|x|}$$

Solution:

$$\lim_{x \to 0^-} \frac{x}{|x|} = \lim_{x \to 0^-} \frac{x}{-x} = \lim_{x \to 0^-} -1 = -1, \qquad \lim_{x \to 0^+} \frac{x}{|x|} = \lim_{x \to 0^+} \frac{x}{x} = \lim_{x \to 0^+} +1 = +1 \quad (3)$$

$$\lim_{x \to 0} \frac{x}{|x|} \qquad \text{D.N.E.} \quad (4)$$

(d) Cancellation-type

$$\lim_{x\to -3} \frac{x^2-9}{x+3}$$

Solution:

$$\lim_{x\to -3} \frac{x^2-9}{x+3} = \lim_{x\to -3} \frac{(x+3)(x-3)}{x+3} = \lim_{x\to -3} \frac{(x-3)}{1} = -3-3 = -6$$

(e) $\frac{\sin\bigcirc}{\bigcirc}$**-type**

$$\lim_{x\to -3} \frac{\sin(x^2-9)}{x+3}$$

Solution:

$$\begin{aligned}\lim_{x\to -3} \frac{\sin(x^2-9)}{x+3} &= \lim_{x\to -3} \frac{\sin(x^2-9)}{x^2-9}\cdot\frac{x^2-9}{x+3}\\ &= \left(\lim_{x\to -3} \frac{\sin(x^2-9)}{x^2-9}\right)\cdot\left(\lim_{x\to -3}\frac{x^2-9}{x+3}\right)\\ &=1\cdot\left(\lim_{x\to -3}\frac{x^2-9}{x+3}\right)\\ &= \lim_{x\to -3}\frac{(x+3)(x-3)}{x+3} = \lim_{x\to -3}\frac{(x-3)}{1} = -3-3=-6\end{aligned}$$

Q3[*Sec1.6, Squeeze Theorem*] Evaluate the following limits

(a)

$$\lim_{x\to 1}(x-1)\cdot\cos\left(\frac{1}{1-x}\right).$$

Solution: $-1 \le \cos\left(\frac{1}{1-x}\right) \le 1$ and $\lim_{x\to 1}(x-1) = 1-1 = 0$ imply that

$$\lim_{x\to 1}(x-1)\cdot\cos\left(\frac{1}{1-x}\right) = 0.$$

(b)

$$\lim_{x\to 0}\sqrt{\frac{x^2}{\cos x+2}}\cdot\sin\left(\frac{1}{x^2}\right)$$

Solution: $-1 \le \sin\left(\frac{1}{x^2}\right) \le 1$ and $\lim_{x\to 0}\sqrt{\frac{x^2}{\cos x+2}} = 0$ imply that

$$\lim_{x\to 0}\sqrt{\frac{x^2}{\cos x+2}}\cdot\sin\left(\frac{1}{x^2}\right) = 0.$$

Q4[*Sec1.8, Domain of continuity*] Use interval notation to indicate where $f(x)$ is continuous.

(a)

$$f(x) = \frac{x^2 - 3x + 1}{x - 3}. \quad \text{Choose from below}$$

A. $(-\infty, +\infty)$; **B**. $(-\infty, 3) \cup (3, +\infty)$; **C**. $(-\infty, 1) \cup (1, +\infty)$; **D**. $(-\infty, 1) \cup (1, 3) \cup (3, +\infty)$.

Solution: $f(x)$ is continuous everywhere in its domain. The domain of $f(x)$ is all those x such that $f(x)$ is computable(meaningful/finite number). The only point not in f's domain is $x = 3$, which makes the denominator zero. Therefore, $f(x)$ is continuous everywhere except $x = 3$. □

(b)

$$f(x) = \sqrt{x + 1}. \quad \text{Choose from below}$$

A. $(-\infty, +\infty)$; **B**. $(-\infty, -1]$; **C**. $[-1, +\infty)$; **D**. $(1, +\infty)$.

Solution: Similar to part (a), $f(x)$ is continuous everywhere in its domain. The expression under square root has to be nonnegative, i.e., $x + 1 \geq 0 \Longrightarrow x \geq -1 \Longrightarrow x \in [-1, +\infty)$. □

(c)

$$f(x) = \frac{(x^2 - 3x + 1)\sqrt{x + 1}}{x - 3}. \quad \text{Use (a,b) to indicate the intervals of continuous for (c)}$$

Solution: The function contains both expression in (a) and (b). Therefore, the domain where $f(x)$ where it is continuous should satisfy both (a) and (b). Combine part (a) and part (b), we have the answer $[-1, 3) \cup (3, +\infty)$. □

Q5[*Sec1.8, Piecewise function*] For what value of k will $f(x)$ be continuous for all values of x ?

$$f(x) = \begin{cases} \frac{x^2 - 3k}{x - 3}, & x \leq 2 \\ 8x - k, & x > 2 \end{cases}$$

Solution: $f(x)$ is a piecewise function which might have a break at the connecting point $x = 2$. The strategy is simply to plug $x = 2$ into the first and second expression of f. Then set them equal and solve for k.

Plug $x = 2$ into $\frac{x^2-3k}{x-3}$, we get $\frac{2^2-3k}{2-3} = \frac{4-3k}{-1} = -(4 - 3k) = 3k - 4$.
Plug $x = 2$ into $8x - k$, we get $8x - k = 8 \cdot 2 - k = 16 - k$.
Set them equal: $3k - 4 = 16 - k \Longrightarrow 4k = 20 \Longrightarrow k = 5$.

The reason why these three steps give us the k such that f is continuous is as follows: $f(x)$ is continuous at $x = 2$ if and only if

$$(*) \quad f(2) = \lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x)$$

It graphically means that the left part of the curve and the right part of the curve are connected at $x = 2$. In the piecewise expression of $f(x)$, it is $\leq$ in the first part. Therefore,

$$f(2) = \lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} \frac{x^2 - 3k}{x - 3} = \frac{4 - 3k}{-1} = 3k - 4$$

Similarly, we have

$$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} 8x - k = 8 \cdot 2 - k = 16 - k$$

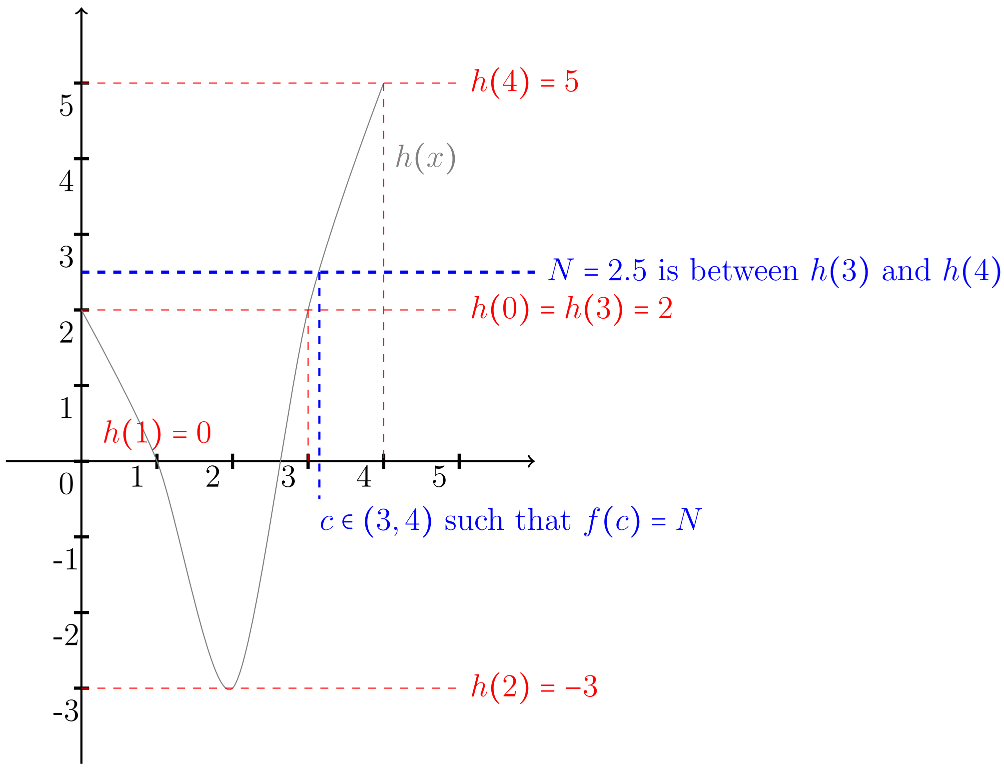
Now due to $(*)$, it is enough to let $3k - 4 = 16 - k$ and solve for k. □

Q6[*Sec1.8, Intermediate Value Theorem(IVT)*] Suppose function $h(x)$ is continuous on $[0,4]$. Suppose $h(0) = 2, h(1) = 0, h(2) = -3, h(3) = 2, h(4) = 5$. For what value of N, the must be a $c \in (3,4)$ such that $h(c) = N$?

A. $N = 0.5$. **B**. $N = 0$. **C**. $N = -2$. **D**. $N = 2.5$.

Options: **A**. $N = 0.5$; **B**. $N = 0$; **C**. $N = -2$; **D**. $N = 2.5$.

Solution: Intermediate Value Theorem(IVT): If f is continuous on $[a,b], f(a) \neq f(b)$, and N is between $f(a)$ and $f(b)$ then there exists $c \in (a,b)$ that satisfies $f(c) = N$.



Q7[*Sec1.8, Intermediate Value Theorem(IVT)*] Let $f(x) = 2x - \cos x$. Prove that there is a solution to the equation $f(x) = 1$, i.e., there exists a number c such that $2c - \cos c = 1$.

Solution: (IVT) $f(x) = 2x - \cos x$ is continuous on $(-\infty, \infty)$ (for all x). It is easy to check that

$$f(0) = 2 \cdot 0 - \cos 0 = -\cos 0 = 1, \qquad f(\frac{\pi}{2}) = 2 \cdot \frac{\pi}{2} - \cos\frac{\pi}{2} = \pi - 0 = \pi \approx 3.14\ldots$$

We want to study the solution to the equation $f(x) = 1$. Clearly, $f(0) = 1 < 1$, $f(\frac{\pi}{2}) = \pi > 1$, i.e., 1 is between $f(0)$ and $f(\frac{\pi}{2})$.

Therefore, according to IVT, there is a c in the interval $(0, \frac{\pi}{2})$ such that $f(c) = 1$, i.e., $2c - \cos c = 1$.

Q8[*Sec2.1/2.2, derivative at given point*] Select all true statements about the function $f(x) = |2x - 4|$

I $\lim_{x\to 0} f(x)$ exists. Yes.

II $f(x)$ is continuous at $x = 0$. Yes.

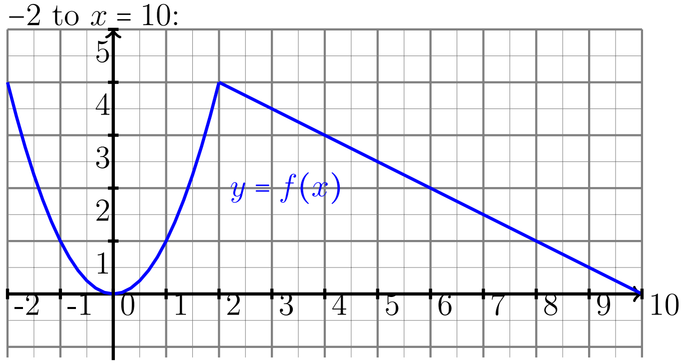
III $f(x)$ is differentiable at $x = 0$. Yes.

IV $\lim_{x\to 2} f(x)$ exists. Yes.

V $f(x)$ is continuous at $x = 2$. Yes.

VI $f(x)$ is differentiable at $x = 2$. No.

Q9[*Sec2.1/2.2, geometric meaning of derivative*] Suppose the graph of $y = f(x)$ is given as follows from $x = -2$ to $x = 10$:



Answer the following questions based on the above graph:

1. Find the open interval(s) where $f'(x) > 0$ and $f'(x) < 0$.

   Solution: $f'(x) > 0$ for x in $(0, 2)$ and $f'(x) < 0$ for x in $(-2, 0)$ and $(2, 10)$.

2. Is $f(x)$ continuous at $x = 2$? Is $f(x)$ differentiable at $x = 2$?

   Solution: It is continuous at $x = 2$ but not differentiable at $x = 2$. The curve has a "sharp turn" at $x = 2$. (The left and right tangent lines are not the same.)

3. Find $f(0)$ and $f'(0)$. Find the equation of the tangent line of $y = f(x)$ at $(0, f(0))$.

   Solution: $f(0) = 0$ and $f'(0) = 0$. The tangent line at $(0, 0)$ is the horizontal axis, $y = 0$.

4. Find $f(6)$ and $f'(6)$. Find the equation of the tangent line of $y = f(x)$ at $(6, f(6))$.

   Solution: From the graph, we can find that $f(6) = 2$ and

   $f'(6) =$ the slope of the tangent line at $x = 6$ =the slope of the straight line from $x = 2$ to $x = 10$

   $$= \frac{f(10) - f(2)}{10 - 2} = \frac{0 - 4}{10 - 2} = -\frac{1}{2}.$$

   (Point-slope formula) equation of the tangent line:

   $$y = \text{slope}\,(x - 6) + f(6) \tag{5}$$

   $$\Longrightarrow y = -\frac{1}{2}(x - 6) + 2 \Longleftrightarrow y = -\frac{1}{2}x + 5 \tag{6}$$

Q10[*Sec2.1/2.2, definition of derivative*] Let $y = \sqrt{x-3}$

(a) ***[Derivative as a limit]*** Use the definition of the derivative to find y'. (Your calculation must include computing a limit.)

Solution:

$$
\begin{aligned}
y' = \lim_{h\to 0} \frac{\sqrt{x+h-3} - \sqrt{x-3}}{h} &= \lim_{h\to 0} \frac{\left(\sqrt{x+h-3} - \sqrt{x-3}\right)}{h} \cdot \frac{\sqrt{x+h-3} + \sqrt{x-3}}{\sqrt{x+h-3} + \sqrt{x-3}} \\
&= \lim_{h\to 0} \frac{\left(\sqrt{x+h-3} - \sqrt{x-3}\right)\left(\sqrt{x+h-3} + \sqrt{x-3}\right)}{h\left(\sqrt{x+h-3} + \sqrt{x-3}\right)} \\
&= \lim_{h\to 0} \frac{(\sqrt{x+h-3})^2 - (\sqrt{x-3})^2}{h\left(\sqrt{x+h-3} + \sqrt{x-3}\right)} \\
&= \lim_{h\to 0} \frac{x+h-3-(x-3)}{h\left(\sqrt{x+h-3} + (x-3)\right)} \\
&= \lim_{h\to 0} \frac{h}{h\left(\sqrt{x+h-3} + (x-3)\right)} \\
&= \lim_{h\to 0} \frac{1}{\sqrt{x+h-3} + \sqrt{x-3}} \\
&= \frac{1}{\sqrt{x+0-3} + \sqrt{x-3}} = \frac{1}{2\sqrt{x-3}}
\end{aligned}
$$

(b) ***[Point-slope formula for the tangent line]*** Find the equation of the tangent line of $y = \sqrt{x-3}$ at $x = 4$.

Solution: At $x = 4$, $y = \sqrt{x-3}\Big|_{x=4} = \sqrt{4-3} = 1$ and

$$y' = \frac{1}{2\sqrt{x-3}}\Bigg|_{x=4} = \frac{1}{2\sqrt{4-3}} = \frac{1}{2}.$$

Point: $(4, 1)$; slope:$\frac{1}{2}$. (Point-slope formula) equation of the tangent line:

$$y = \frac{1}{2}(x-4) + 1 \Longleftrightarrow y = \frac{1}{2}x - 1.$$

Q11[*Sec2.3/2.4/2.5, Differentiation Formulas/Laws*] Find the derivatives of the following functions. Do not need to simplify.

(a)[Linear Rule+Power functions]

$T(x) = 2\sqrt{x} - \dfrac{1}{2\sqrt{x}}$

Solution:

$$T'(x) = \left(2\sqrt{x} - \frac{1}{2\sqrt{x}}\right)' = \left(2x^{\frac{1}{2}}\right)' - \left(\frac{1}{2}x^{-\frac{1}{2}}\right)' = 2\cdot\frac{1}{2}x^{\frac{1}{2}-1} - \frac{1}{2}\cdot(-\frac{1}{2})x^{-\frac{1}{2}-1} = x^{-\frac{1}{2}} + \frac{1}{4}x^{-3/2}$$

(b)[Product Rule+Power functions]

$g(t) = \big(-1 + 2t\big)\big(\sin t + 2\big)$

Solution:

$$\begin{aligned} g'(t) &= (-1 + 2t)'\big(\sin t + 2\big) + \big(-1 + 2t\big)(\sin t + 2)' \\ &= (0 + 2)\big(\sin t + 2\big) + \big(-1 + 2t\big)(\cos t + 0) = 2\big(\sin t + 2\big) + \big(-1 + 2t\big)\cos t \end{aligned}$$

(c)[Trig functions+Chain Rule]

$y = \sin(x^2 + 1)$

Solution: Outer function: $\sin(\blacksquare), (\sin(\blacksquare))' = \cos(\blacksquare)$; Inner function: $x^2 + 1, inner' = (x^2 + 1)' = 2x$.

$$y' = (\sin(x^2 + 1))' = outer'(inner)\cdot inner' = \cos(x^2 + 1)\cdot(2x)$$

(d)[Quotient Rule+Trig functions+Chain Rule]

$f(t) = \dfrac{3t}{\tan(t^2 - 1)}$

Solution: (quotient rule first):

$$f'(t) = \left(\frac{3t}{\tan(t^2 - 1)}\right)' = \frac{(3t)'\cdot\tan(t^2 - 1) - 3t\cdot(\tan(t^2 - 1))'}{(\tan(t^2 - 1))^2} = \frac{3\cdot\tan(t^2 - 1) - 3t\cdot(\tan(t^2 - 1))'}{(\tan(t^2 - 1))^2}$$

(Chain Rule:) $(\tan(t^2 - 1))' = outer'(inner)\cdot inner' = \sec^2(t^2 - 1)\cdot(2t)$.

$$f'(t) = \left(\frac{3t}{\tan(t^2 - 1)}\right)' = \frac{3\cdot\tan(t^2 - 1) - 3t\cdot(\tan(t^2 - 1))'}{(\tan(t^2 - 1))^2} = \frac{3\cdot\tan(t^2 - 1) - 3t\cdot\sec^2(t^2 - 1)\cdot(2t)}{(\tan(t^2 - 1))^2}$$

(e)[Trig functions+Double Chain Rule]

$f(x) = 3\sec\big(\cos(2x)\big)$

1st Chain rule: Outer function: $3\sec(\blacksquare)$; Inner function: $\cos(2x)$.

$$f'(x) = outer'(inner)\cdot inner' = 3\sec(\cos(2x))\cdot\tan(\cos(2x))\cdot(\cos(2x))'$$

2nd Chain rule: $(\cos(2x))' = -\sin(2x)\cdot 2$. Put these two together, we have

$$\begin{aligned} f'(x) = outer'(inner)\cdot inner' &= 3\sec(\cos(2x))\cdot\tan(\cos(2x))\cdot(-\sin(2x)\cdot 2) \\ &= -6\sec(\cos(2x))\cdot\tan(\cos(2x))\cdot\sin(2x) \end{aligned}$$

Q12[*Sec2.7, Rates of Change/Functions of motion*] The height of a projectile is given by the function $h(t) = -4t^2 + 8t + 40$, where t is measured in seconds and h in feet.

(a)[**Velocity and position**] Find the velocity $v(t)$ at time t.

Solution:

$$v(t) = h'(t) = (-4t^2 + 8t + 40)' = -4 \cdot 2t + (8t)' + (40)' = -8t + 8$$

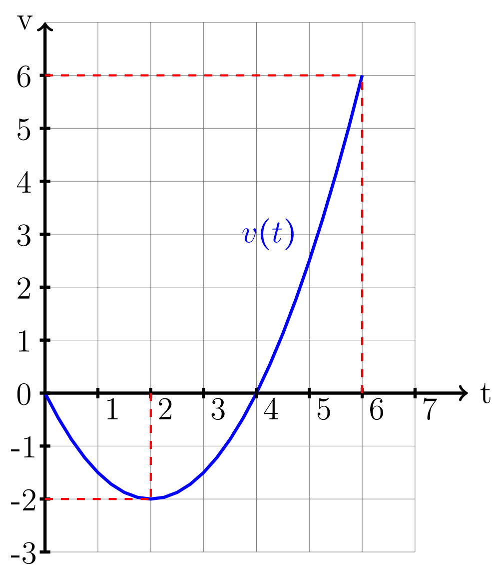
(b) Find the maximum height of the projectile?

Solution: Maximum height is reached when the velocity is zero. Set $v(t) = -8t + 8 = 0$ and solve for $t = 1$ (s). Then the maximum height= $h(1) = -4 \cdot 1^2 + 8 \cdot 1 + 40 = 44$ (feet).

(c)[**Acceleration and velocity**] What is the acceleration $a(6)$ after 6 seconds?

Solution: $a(t) = v'(t) = (-8t + 8)' = -8$ (for all t). Then $a(6) = -8$ (ft/s^2).

Q13[*Sec2.7, Graph of the velocity*] The accompanying figure shows the velocity $v(t)$ of a particle moving on a horizontal coordinate line, for t in the closed interval $[0, 6]$.



Solution:

(a) When does the particle move forward?

Move forward $\iff v > 0 \iff t \in (4, 6)$

(b) When does the particle slow down?

Slow down $\iff$ Speed $|v|$ drops $\iff t \in (2, 4)$

(c) When is the particle's acceleration positive?

acceleration positive $\iff a(t) = v'(t) > 0 \iff$ slope of the tangent line is positive/v is increasing $\iff t \in (2, 6)$

(d) When does the particle move at its greatest speed in $[0, 6]$?

greatest speed $\iff$ highest or lowest point in the graph $\iff t = 6$ (greatest speed=6)

Q14[*Sec2.6, Implicit differentiation*] Consider the curve $y^2 + 2xy + x^3 = x$

(a) Find $\frac{dy}{dx}$ in terms of x, y. Apply Implicit differential rule to the equation $y^2 + 2xy + x^3 = x$.

$$\begin{aligned} (y^2 + 2xy + x^3)' &= (x)' \\ \Longrightarrow (y^2)' + (2xy)' + (x^3)' &= 1 \qquad (*) \\ \Longrightarrow 2y \cdot y' + 2y + 2xy' + 3x^2 &= 1 \qquad (**) \end{aligned}$$

From $(*)$ to $(**)$, we use the chain rule for $(y^2)'$ and product rule for $(2xy)'$, where

$$\begin{aligned} &\text{chain rule} : (y^2)' = 2y(x) \cdot y'(x) = 2yy' \\ &\text{product rule} : (2xy)' = (2x)' \cdot y(x) + 2x \cdot y'(x) = 2y + 2xy' \\ &(x^3)' = 3x^2 \end{aligned}$$

$(**)$: leave all the terms containing y' on the left hand side and move all the rest terms to the right hand side of the equation, and then solve for y':

$$\begin{aligned} &2y \cdot y' + 2y + 2x \cdot y' + 3x^2 = 1 \\ \Longrightarrow &2y \cdot y' + 2x \cdot y' = 1 - 2y - 3x^2 \\ \Longrightarrow &(2y + 2x) \cdot y' = 1 - 2y - 3x^2 \\ \Longrightarrow &\frac{dy}{dx} = y' = \frac{1 - 2y - 3x^2}{2y + 2x} \end{aligned}$$

(b) Find $\frac{dy}{dx}$ at $(1, -2)$ and find the slope of the tangent line of the curve at the point $(1, -2)$. Plug $(x, y) = (1, -2)$ into the expression in part (a), we have

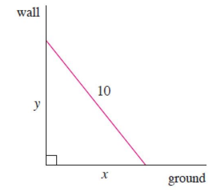
$$\begin{aligned} \frac{dy}{dx} = y' = \frac{1 - 2y - 3x^2}{2y + 2x} &= \frac{1 - 2 \times (-2) - 3 \times 1^2}{2 \times (-2) + 2 \times 1} \\ &= \frac{1 + 4 - 3}{-4 + 2} \\ &= \frac{2}{-2} \\ &= -1 \end{aligned}$$

(c) Find the equation of the tangent line of the curve at the point $(1, -2)$.

Solution: Slope=-1. Point (1,-2). The slope-point formula gives the formula for the tangent line:

$$y = (-1)(x - 1) - 2 \Longleftrightarrow y = -x - 1$$

Q15, *Sec2.8, Related Rates* A ladder 10 ft long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of 1 ft/s, how fast is the top of the ladder sliding down the wall when the bottom of the ladder is 6 ft from the wall?



Solution: Pythagorean theorem: $x^2 + y^2 = 10^2$, where $x = x(t)$, $y = y(t)$ are both functions of t. Take derivative with respect to t both sides of the equations:

$$(x^2 + y^2)' = (10^2)' \iff (x^2)' + (y^2)' = (10^2)'$$
$$\iff 2x \cdot x' + 2y \cdot y' = 0$$

From the problem, we know that $x' = 1$ ft/s and $x = 6$. At that moment, we can solve for y from $6^2+y^2 = 10^2$, which gives $y^2 = 100 - 36 = 64 \Longrightarrow y = 8$ ft. Then plug $x' = 1, x = 6, y = 8$ into $2x \cdot x' + 2y \cdot y' = 0$, we have

$$2 \cdot 6 \cdot 1 + 2 \cdot 8 \cdot y' = 0 \Longrightarrow y' = -\frac{3}{4} \text{ (ft/s)}.$$

(Remark: the negative sign of y' means that y is decreasing at a rate of $\frac{3}{4}$ ft/s.) □

Q16, Challenging problem The gas law for an ideal gas at absolute temperature T(in kelvins=K), pressure P(in atmospheres=atm), and volume V(in liters=L) is given by

$$P = \frac{nRT}{V},$$

where n is the number of moles of the gas (constant) and R is the gas constant.

(a) Suppose n, R, V are all constants. Find the rate of change of the pressure with respect to the temperature $\frac{dP}{dT}$.

Solution: Since $\frac{nR}{V}$ is a constant, take derivative with respect to T gives that

$$P = \frac{nRT}{V} = \frac{nR}{V} \cdot T \Longrightarrow \frac{dP}{dT} = \frac{d\left(\frac{nR}{V} \cdot T\right)}{dT} = \frac{nR}{V}$$

□

(b) Suppose n, R, T are all constants. Find the rate of change of the pressure with respect to the volume $\frac{dP}{dV}$.

Solution: Since nRT is a constant, take derivative with respect to V gives that

$$P = \frac{nRT}{V} = nRT \cdot V^{-1} \Longrightarrow \frac{dP}{dV} = \frac{d\left(nRT \cdot V^{-1}\right)}{dV} = nRT \cdot \frac{d\left(V^{-1}\right)}{dV} = nRT \cdot (-V^{-2}) = -\frac{nRT}{V^2}$$

□

(c) Suppose the rate of change of the pressure with respect to the volume is -0.10 atm/L when the volume of the gas is 2 L. Find the the rate of change of the pressure with respect to the volume when the volume of the gas is 4 L.

Solution: When $V = 2$, by part (b),

$$-0.10 = \frac{dP}{dV} = -\frac{nRT}{V^2} = -\frac{nRT}{2^2}$$

We can solve for nRT (as an entire piece) as $nRT = 4 \times 0.10 = 0.40$. Therefore, if $V = 4$, then (plug $nRT = 0.40$ entirely)

$$\frac{dP}{dV} = -\frac{nRT}{V^2} = -\frac{0.40}{4^2} = -0.025 \text{ (atm/L)}.$$

□