Practice Mid 1, Sec39. Fall 2018

Q1[*Sec1.4, Average rate of change/Average velocity, see also Q9*] Let $f(x) = \cos x + 2$. Compute the average rate of change of $f(x)$ on the interval $[0, \frac{\pi}{2}]$

Q2[*Sec1.5/1.6, Limit and Limit Laws*] Evaluate the following limits

(a)Direct plug in-type

$$\lim_{x \to 0} \sqrt{\frac{x^2}{\cos x + 2}}$$

(b)$\frac{1}{0}$-type/One-sided limits

$$\lim_{x \to 0^+} \frac{x-3}{x(x+5)} \qquad \lim_{x \to 0^-} \frac{x-3}{x(x+5)} \qquad \lim_{x \to 0} \frac{x-3}{x(x+5)}$$

(c)Absolute value

$$\lim_{x \to 0^-} \frac{x}{|x|} \qquad \lim_{x \to 0^+} \frac{x}{|x|} \qquad \lim_{x \to 0} \frac{x}{|x|}$$

(d) Cancellation-type

$$\lim_{x \to -3} \frac{x^2 - 9}{x + 3}$$

(e) $\frac{\sin \bigcirc}{\bigcirc}$-type

$$\lim_{x \to -3} \frac{\sin(x^2 - 9)}{x + 3}$$

Q3[*Sec1.6, Squeeze Theorem*] Evaluate the following limits

(a)

$$\lim_{x \to 1} (x - 1) \cdot \cos\left(\frac{1}{1 - x}\right).$$

(b)

$$\lim_{x \to 0} \sqrt{\frac{x^2}{\cos x + 2}} \cdot \sin\left(\frac{1}{x^2}\right)$$

Q4[*Sec1.8, Domain of continuity*] Use interval notation to indicate where $f(x)$ is continuous.

(a)

$$f(x) = \frac{x^2 - 3x + 1}{x - 3}. \quad \text{Choose from below}$$

A. $(-\infty, +\infty)$; **B**. $(-\infty, 3) \cup (3, +\infty)$; **C**. $(-\infty, 1) \cup (1, +\infty)$; **D**. $(-\infty, 1) \cup (1, 3) \cup (3, +\infty)$.

(b)

$$f(x) = \sqrt{x + 1}. \quad \text{Choose from below}$$

A. $(-\infty, +\infty)$; **B**. $(-\infty, -1]$; **C**. $[-1, +\infty)$; **D**. $(1, +\infty)$.

(c)

$$f(x) = \frac{(x^2 - 3x + 1)\sqrt{x + 1}}{x - 3}. \quad \text{Use (a),(b) to indicate the intervals of continuity for (c).}$$

Q5[*Sec1.8, Piecewise function*] For what value of k will $f(x)$ be continuous for all values of x ?

$$f(x) = \begin{cases} \frac{x^2 - 3k}{x - 3}, & x \leq 2 \\ 8x - k, & x > 2 \end{cases}$$

Q6[*Sec1.8, Intermediate Value Theorem(IVT)*] Suppose function $h(x)$ is continuous on $[0, 4]$. Suppose $h(0) = 2, h(1) = 0, h(2) = -3, h(3) = 2, h(4) = 5$. For what value of N, the must be a $c \in (3, 4)$ such that $h(c) = N$?

A. $N = 0.5$. **B**. $N = 0$. **C**. $N = -2$. **D**. $N = 2.5$.

Q7[*Sec1.8, Intermediate Value Theorem(IVT)*] Let $f(x) = 2x - \cos x$. Prove that there is a solution to the equation $f(x) = 1$, i.e., there exists a number c such that $2c - \cos c = 1$.

Q8[*Sec2.1/2.2, derivative at given point*] Select all true statements about the function $f(x) = |2x - 4|$

I $\lim_{x \to 0} f(x)$ exists.

II $f(x)$ is continuous at $x = 0$.

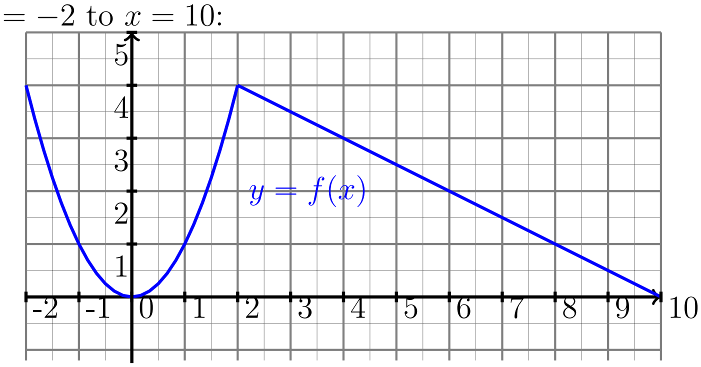
III $f(x)$ is differentiable at $x = 0$.

IV $\lim_{x \to 2} f(x)$ exists.

V $f(x)$ is continuous at $x = 2$.

VI $f(x)$ is differentiable at $x = 2$.

Q9[*Sec2.1/2.2, geometric meaning of derivative*] Suppose the graph of $y = f(x)$ is given as follows from $x = -2$ to $x = 10$:



Answer the following questions based on the above graph:

1. Find the open interval(s) where $f'(x) > 0$ and $f'(x) < 0$.

2. Is $f(x)$ continuous at $x = 2$? Is $f(x)$ differentiable at $x = 2$?

3. Find $f(0)$ and $f'(0)$. Find the equation of the tangent line of $y = f(x)$ at $(0, f(0))$.

4. Find $f(6)$ and $f'(6)$. Find the equation of the tangent line of $y = f(x)$ at $(6, f(6))$.

Q10[*Sec2.1/2.2, definition of derivative*] Let $y = \sqrt{x-3}$

(a) ***[Derivative as a limit]*** Use the definition of the derivative to find y'. (Your calculation must include computing a limit.)

(b) ***[Point-slope formula for the tangent line]*** Find the equation of the tangent line of $y = \sqrt{x-3}$ at $x = 4$.

Q11[*Sec2.3/2.4/2.5, Differentiation Formulas/Laws*] Find the derivatives of the following functions. Do not need to simplify.

(a)[Linear Rule+Power functions]

$$T(x) = 2\sqrt{x} - \frac{1}{2\sqrt{x}}$$

(b)[Product Rule+Power functions]

$$g(t) = (-1 + 2t)(\sin t + 2)$$

(c)[Trig functions+Chain Rule]

$$y = \sin(x^2 + 1)$$

(d)[Quotient Rule+Trig functions+Chain Rule]

$$f(t) = \frac{3t}{\tan(t^2 - 1)}$$

(e)[Trig functions+Double Chain Rule]

$$f(x) = 3\sec(\cos(2x))$$

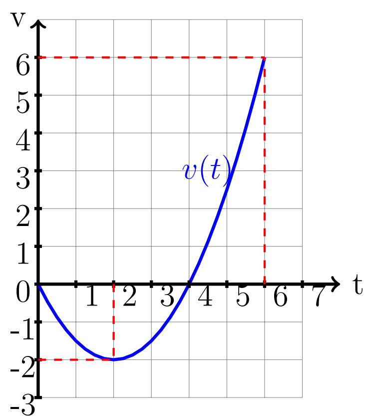
Q12[*Sec2.7, Rates of Change/Functions of motion*] The height of a projectile is given by the function $h(t) = -4t^2 + 8t + 40$, where t is measured in seconds and h in feet.

(a)[**Velocity and position**] Find the velocity $v(t)$ at time t.

(b) Find the maximum height of the projectile?

(c)[**Acceleration and velocity**] What is the acceleration $a(6)$ after 6 seconds?

Q13[*Sec2.7, Graph of the velocity*] The accompanying figure shows the velocity $v(t)$ of a particle moving on a horizontal coordinate line, for t in the closed interval $[0, 6]$.



(a) When does the particle move forward?

(b) When does the particle slow down?

(c) When is the particle's acceleration positive?

(d) When does the particle move at its greatest speed in $[0, 6]$?

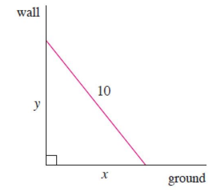
Q14[*Sec2.6, Implicit differentiation*] Consider the curve $y^2 + 2xy + x^3 = x$

(a) Find $\frac{dy}{dx}$ in terms of x, y.

(b) Find $\frac{dy}{dx}$ at $x = 1$ and find the slope of the tangent line of the curve at the point $(1, -2)$.

(c) Find the equation of the tangent line of the curve at the point $(1, -2)$.

Q15, *Sec2.8, Related Rates* A ladder 10 ft long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of 1 ft/s, how fast is the top of the ladder sliding down the wall when the bottom of the ladder is 6 ft from the wall?



Q16, Challenging problem The gas law for an ideal gas at absolute temperature T(in kelvins=K), pressure P(in atmospheres=atm), and volume V(in liters=L) is given by

$$P = \frac{nRT}{V},$$

where n is the number of moles of the gas (constant) and R is the gas constant.

(a) Suppose n, R, V are all constants. Find the rate of change of the pressure with respect to the temperature $\frac{\mathrm{d}P}{\mathrm{d}T}$.

(b) Suppose n, R, T are all constants. Find the rate of change of the pressure with respect to the volume $\frac{\mathrm{d}P}{\mathrm{d}V}$.

(c) Suppose the rate of change of the pressure with respect to the volume is -0.10 atm/L when the volume of the gas is 2 L. Find the the rate of change of the pressure with respect to the volume when the volume of the gas is 4 L.