Name: ______________________ **ID:** ______________

Clear your desk of everything except pens, pencils and erasers. Show all work clearly and in order. No notes, phones and calculators. You have 10 minutes to finish these TWO problems for 10 points.
Formula Sheet.

- nth term test for divergence: If $\lim_{n\to\infty} a_n$ does not exist or if $\lim_{n\to\infty} a_n \neq 0$, then the series $\sum_{n=1}^{\infty} a_n$ is divergent.
- The p-series: $\sum_{n=1}^{\infty} \frac{1}{n^p}$ is convergent if $p > 1$ and divergent if $p \leq 1$.
- Geometric: If $|r| < 1$, then $\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}$

1. (6 points) Determine whether the following SERIES is convergent or divergent, state the reason (test).

- $$\sum_{n=1}^{\infty} \frac{2n+1}{3n+2}$$

  According to test for DIV,

  $\lim_{n\to\infty} \frac{2n+1}{3n+2} \underset{\text{Terms}}{\overset{\text{leading}}{=}} \lim_{n\to\infty} \frac{2\cancel{n}}{3\cancel{n}} = \frac{2}{3} \neq 0.$ $\sum \frac{2n+1}{3n+2}$ is divergent.

- $$\sum_{n=1}^{\infty} \frac{3\sqrt{n}}{n^2}$$

  $a_n = \frac{3\sqrt{n}}{n^2} = 3 \cdot \frac{n^{\frac{1}{2}}}{n^2} = 3 \cdot \frac{1}{n^{\frac{3}{2}}}$, $p = \frac{3}{2} > 1.$

  According to p-Series test,

  $\sum \frac{3\sqrt{n}}{n^2}$ is Convergent.

- $$\sum_{n=0}^{\infty} (-\frac{3}{2})^n$$

  Geometric Series, where $r = \frac{-3}{2}$. $(a=1)$.

  $|r| = \left|\frac{-3}{2}\right| = \frac{3}{2} > 1.$

  So the Geometric Series $\sum (-\frac{3}{2})^n$ is divergent

2. (4 points) Find the sum of the series

$$\sum_{n=0}^{\infty} \frac{3-2^n}{7^n}$$

$$= \sum_{n=0}^{\infty} \frac{3}{7^n} - \sum_{n=0}^{\infty} \frac{2^n}{7^n}$$

$$\sum_{n=0}^{\infty} \frac{3}{7^n} = \sum_{n=0}^{\infty} 3 \cdot \left(\frac{1}{7}\right)^n, \quad a=3,\ r=\frac{1}{7}.$$

$$= \frac{a}{1-r} = \frac{3}{1-\frac{1}{7}} = \frac{3}{\frac{6}{7}} = 3 \cdot \frac{7}{6} = \frac{7}{2}$$

$$\sum_{n=0}^{\infty} \frac{2^n}{7^n} = \sum_{n=0}^{\infty} 1 \cdot \left(\frac{2}{7}\right)^n, \quad a=1,\ r=\frac{2}{7}$$

$$= \frac{a}{1-r} = \frac{1}{1-\frac{2}{7}} = \frac{1}{\frac{5}{7}} = \frac{7}{5}$$

$$\Rightarrow \sum_{n=0}^{\infty} \frac{3-2^n}{7^n} = \frac{7}{2} - \frac{7}{5} = \frac{35-14}{10} = \boxed{\frac{21}{10}}$$