Name: _____ **ID:** _____

Clear your desk of everything except pens, pencils and erasers. Show all work clearly and in order. No notes, phones and calculators. You have 10 minutes to finish these two problems for 10 points.

Trig Identities: $\sin^2 x + \cos^2 x = 1$.

Integration by Parts: $\int u\, dv = uv - \int v\, du$

1. (4 points) Evaluate the following integral:

$$\int \ln(\sqrt{t})\, dt \quad \text{(Hint: Simplify the function } \ln(\sqrt{t}) \text{ first).}$$

$= \int \frac{1}{2}\ln t \cdot dt$. $u = \frac{1}{2}\ln t$, $dv = dt$

$du = \frac{1}{2}\cdot\frac{1}{t}dt$, $v = t$

$= \frac{1}{2}\ln t \cdot t - \int t \cdot \frac{1}{2}\frac{1}{t} dt$

$= \boxed{\frac{1}{2}\ln t \cdot t - \frac{1}{2}t + C}$

2. (6 points) Evaluate the integral, your final answer should NOT contain any TRIGONOMETRIC FUNCTION:

$$\int \frac{x^3}{\sqrt{1-x^2}}\, dx$$

$x = \sin\theta$, $dx = \cos\theta\, d\theta$

$= \int \frac{\sin^3\theta}{\sqrt{1-\sin^2\theta}} \cdot \cos\theta \cdot d\theta$

$= \int \frac{\sin^3\theta}{\cos\theta} \cdot \cos\theta\, d\theta$

$= \int \sin^3\theta \cdot d\theta$

$= \int \sin^2\theta \sin\theta\, d\theta$

$= \int (1-u^2)\cdot(-du)$.

ADD rule

$u = \cos\theta$, $du = -\sin\theta\, d\theta$

$\sin^2\theta = 1 - \cos^2\theta = 1 - u^2$

$= \int u^2 - 1\, du$

$= \frac{1}{3}u^3 - u$

$= \frac{1}{3}\cos^3\theta - \cos\theta$

$\cos\theta = \sqrt{1-\sin^2\theta} = \sqrt{1-x^2}$

$= \boxed{\frac{1}{3}\left(\sqrt{1-x^2}\right)^3 - \sqrt{1-x^2} + C}$