

Name: _____

ID: _____

Clear your desk of everything except pens, pencils and erasers. Show all work clearly and in order. No notes, phones and calculators. You have 10 minutes to finish these **TWO** problems for 10 points.

Formula Sheet.

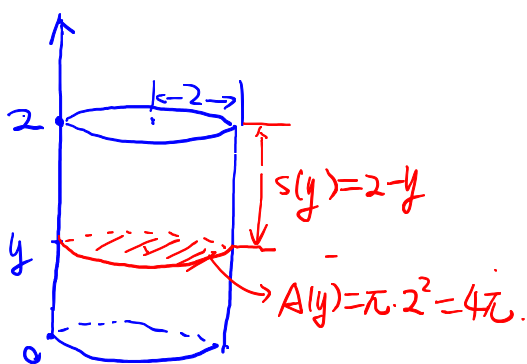
- Work: Suppose $f(x)$ is a force function. The work in moving an object from a to b is given by:

$$W = \int_a^b f(x) dx$$
- If f is a one-to-one differentiable function with inverse function f^{-1} and $f'(f^{-1}(a)) \neq 0$, then the inverse function is differentiable at a and

$$(f^{-1})'(a) = \frac{1}{f'(f^{-1}(a))}$$

1. A vertical right cylindrical tank has **height 2 ft** and **radius 2 ft**. It is full of soda **weighing 60 lbs/ft³**. How much work does it take to pump all of the soda from a tank to an outlet which is at **the level of the top of the tank**.

- (a) (5 points) Set up the integral for the work.



$$V = \int_0^2 60 \cdot (2-y) \cdot 4\pi \cdot dy$$

- (b) (2 points) Evaluate the integral.

$$\begin{aligned} V &= 240\pi \cdot \int_0^2 (2-y) \cdot dy = 240\pi \cdot \left(2y - \frac{1}{2}y^2 \right) \Big|_0^2 \\ &= 240\pi \cdot (4-2) = 480\pi \text{ ft}\cdot\text{lb.} \end{aligned}$$

2. (3 points) Suppose that the differentiable function $y = f(x)$ has an inverse. The graph of f passes through the origin with slope 4, i.e., $f(0) = 0$ and $f'(0) = 4$. Find the slope of the graph of f^{-1} at the origin.

$$f'(0) = 4, \quad f(0) = 0 \Rightarrow f^{-1}(0) = 0$$

$$(f^{-1})'(0) = \frac{1}{f'(f^{-1}(0))} = \frac{1}{f'(0)} = \boxed{\frac{1}{4}}$$

