Practice Midterm2, Sec61, F17

Sec8.1. Arc-length. LecNote6.

Q1 Find the exact arc-length of $x = \frac{2}{3}(y^2+1)^{3/2}$ from $y = 0$ to $y = 2$.

Sec11.1. Sequences. LecNote7.

Q2(Limit of a sequence.) Find the limit if the sequence below converges or state why it diverges.

(a)

$$a_n = \frac{1}{n} \ln(\frac{1}{n})$$

(b)

$$a_k = \frac{\sqrt{1+k^3}}{3k^2+7k}$$

(c)

$$a_n = n(e^{\frac{1}{n}} - 1)$$

Sec11.2. (Introduction to) Series. LecNote7.

Q3 (n-th term test for divergence). Which statements (more than one option) are true about

$$a_n = e^{\frac{2}{n}}, \quad \text{and} \quad \sum_{n=1}^{\infty} e^{\frac{2}{n}}$$

A. The sequence a_n is convergent but the series is divergent.

B. The sequence a_n converges, therefore, the **nth term test** concludes that the series converges.

C. The sequence a_n has limit zero, therefore, the **nth term test** concludes that the series converges.

D. The **nth term test** concludes that the series diverges.

E. If the series is convergent, then the sequence a_n should have zero limit, which is a contradiction. Therefore, the series can not be convergent.

Q4 (Geometric Sequence/Sum/Series). Find the sum of the series

$$\sum_{n=1}^{\infty} \frac{9^{n/2}}{3(2^{2n+1})}$$

Sec11.3. Integral Test and the p-Series. LecNote7.

Q5 (Integral Test) Test the following series for convergence or divergence by THE INTEGRAL TEST.

$$\sum_{n=1}^{\infty} \frac{1}{n^2} \sin\left(\frac{1}{n}\right)$$

Q6 (p-series) Which statements (more than one option) are true

A. The series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ is convergent if and only if $|p| < 1$.

B. The series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ is divergent if and only if $|p| > 1$.

C. The series $\sum_{n=1}^{\infty} r^n$ is convergent if $|r| < 1$.

D. The series $\sum_{n=0}^{\infty} r^n$ is divergent at $r = -1$.

E. The series $\sum_{n=1}^{\infty} \frac{-3\sqrt{n}}{n^{1.5}}$ is convergent since it is a constant multiple of a p-series with $p = 1.5 > 1$.

F. The p-series $\sum_{n=1}^{\infty} n^{-2}$ is divergent since $p = -2$ and $|p| = 2 > 1$.

Sec11.4. Comparison Test. LecNote8.

Q7 Determine whether the following series converge or diverge by (Direct/Limit) Comparison Test.

(a)

$$\sum_{n=1}^{\infty} \frac{\sqrt{n+1}}{n}$$

(b)

$$\sum_{n=1}^{\infty} \frac{\sqrt{n^2+1}}{3n^3 - 7n}$$

(c)

$$\sum_{n=2}^{\infty} \frac{2}{n^{61}+1}$$

Sec11.5. Alternating Series Test and Absolute Convergence. LecNote8.

Q8 Determine whether the following series are absolutely convergent, conditionally convergent, or divergent:

(a)

$$(1)\quad \sum_{n=1}^{\infty} \frac{\cos(5n)}{n^5} \qquad \text{and } (2) \quad \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{3n}$$

A (1) is absolutely convergent; (2) is divergent.

B (1) is conditionally convergent; (2) is divergent.

C (1) is absolutely convergent; (2) is conditionally convergent.

D (1) is divergent; (2) is conditionally convergent.

E (1) and (2) are conditionally convergent.

(b) Determine whether the following series are absolutely convergent, conditionally convergent, or divergent:

$$(1)\quad \sum_{n=1}^{\infty} \frac{\sin(n)+1}{2^n} \qquad \text{and } (2) \quad \sum_{n=1}^{\infty} \frac{(-1)^{n}}{\sqrt{n}}$$

A (1) is absolutely convergent; (2) is divergent.

B (1) is conditionally convergent; (2) is divergent.

C (1) is absolutely convergent; (2) is conditionally convergent.

D (1) is divergent; (2) is conditionally convergent.

E (1) and (2) are conditionally convergent.

Sec11.6. Ratio Test. LecNote8.

Q9 Determine whether the following series converge or diverge.

(a)

$$\sum_{n=1}^{\infty} \frac{2^n(n^2+1)}{3^n}$$

(b)

$$\sum_{n=1}^{\infty} \frac{(n+1)\,!}{e^n}$$

(c)

$$\sum_{n=1}^{\infty} \frac{n}{(-2)^n}$$

Sec11.8. Power Series. LecNote9.

Q10 Consider the following power series

$$\sum_{n=0}^{\infty}(n+3)\left(\frac{2x-3}{3}\right)^n$$

(a) Find the radius of convergence of the series and the OPEN interval of the convergence.

(b) Test the Left and Right Endpoints of the open interval in Part (a) for convergence or divergence.

Sec11.9. Power Series Representation. LecNote9.

Q11 Consider the function $f(x) = \frac{x}{1+x}$

(a) Find the first FOUR non-zero terms of the power series representation of the function $f(x)$

(b) Use the expression in Part (a) to find the first THREE non-zero terms of the power series representation of the DERIVATIVE function of f,

$$f'(x) = \sum_{n=0}^{\infty} c_n x^n$$

(c) Use the expression in Part (a) to find the first THREE non-zero terms of the power series representation of the indefinite INTEGRAL of f,

$$\int f(x)dx = \sum_{n=0}^{\infty} c_n x^n + C$$

Sec11.10/11. Taylor and Maclaurin Series. LecNote10. **Q12** Find the 3rd degree Taylor polynomial of $f(x) = x\cos(2x)$ centered at $x = \pi/6$

Q13 Consider the function $f(x) = \ln(1 - 2x)$.

(a) Find the Maclaurin series of $f(x)$.

(b) 'Evaluate'(Guess) the limit $\lim_{x\to 0} \frac{\ln(1-2x)}{x}$ by the power series expression in Part (a).

(c) Verify the answer in (b) by l'Hopital's Rule.

Q14 Consider the function $f(x) = xe^x$.

(a) Find the power series expansion of the function $f(x) = xe^x$ centered at $x = 0$.

(b) Find the 3rd degree Taylor polynomial $T_3(x)$ of $f(x)$ at $x = 0$. Find $T_3(0.5)$ for estimating $f(0.5)$.

(c) Suppose we know that $|f^4(x)| \leq 15$ for all $|x| \leq 1$, what the error in Part (b) when we use $T_3(0.5)$ to approximate $f(0.5)$? And what's the maximal error for estimating $f(x)$ via $T_3(x)$ on $[-1, 1]$.