Name: ______________________________

Section: ________________ **Recitation Instructor**: ______________________________

READ THE FOLLOWING INSTRUCTIONS.

- **Do not open your exam until told to do so.**
- No calculators, cell phones or any other electronic devices can be used on this exam.
- Clear your desk of everything excepts pens, pencils and erasers.
- If you need scratch paper, use the back of the previous page.
- Without fully opening the exam, check that you have pages 1 through 11.
- Fill in your name, etc. on this first page.
- **Show all your work.** Write your answers clearly! Include enough steps for the grader to be able to follow your work. Don't skip limits or equal signs, etc. Include words to clarify your reasoning.
- Do first all of the problems you know how to do immediately. Do not spend too much time on any particular problem. Return to difficult problems later.
- If you have any questions please raise your hand and a proctor will come to you.
- There is no talking allowed during the exam.
- You will be given exactly 90 minutes for this exam.
- Remove and utilize the formula sheet provided to you at the end of this exam.
- This is a practice exam. The actual exam may differ significantly from this practice exam because there are many varieties of problems that can test each concept.

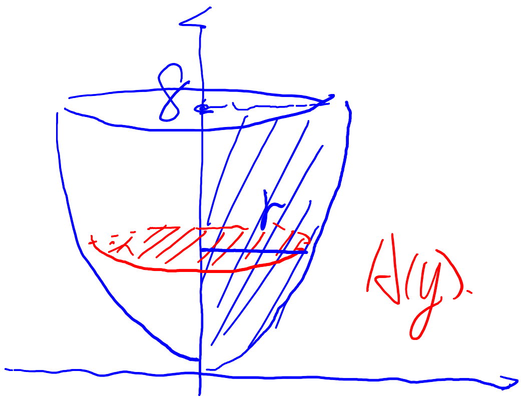
I have read and understand the above instructions: ______________________________

SIGNATURE

Standard Response Questions. Show all work to receive credit. Please $\boxed{\textbf{BOX}}$ your final answer.

1. Consider the region in the xy-plane above the curve $y = x^3$, below the line $y = 8$, and to the right of the y-axis.

   (a) (7 points) Sketch this region, and compute the volume of the solid obtained by rotating this region around the y-axis



$$y=x^3 \Leftrightarrow r=x=y^{\frac{1}{3}}$$

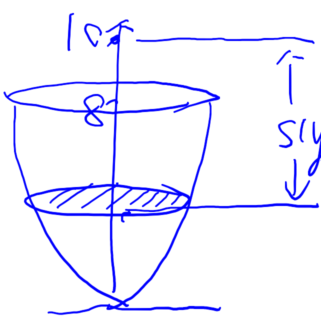
$$V=\int_0^8 \pi\cdot r^2\cdot dy=\int_0^8 \pi\cdot y^{\frac{2}{3}}\,dy$$

$$=\pi\cdot\frac{1}{\frac{2}{3}+1}\,y^{\frac{5}{3}}\Big|_0^8$$

$$=\pi\cdot\frac{3}{5}\cdot 8^{\frac{5}{3}}$$

$$=\pi\cdot\frac{3}{5}\left(8^{\frac{1}{3}}\right)^5=\pi\cdot\frac{3}{5}\cdot 2^5$$

   (b) (7 points) Assume the solid in part (a) is a storage tank (with all lengths in feet), full of a liquid weighing 4 pounds per cubic foot. Compute the work (in ft-lbs) needed to pump all the liquid to the 2 feet above of the top of the tank.



$s(y)=10-y$, $\sigma=4$. $A(y)=\pi\cdot r^2=\pi\cdot y^{\frac{2}{3}}$

$$W=\int_0^8 \sigma\cdot s(y)\cdot A(y)\,dy$$

$$=\int_0^8 4\cdot(10-y)\cdot\pi\cdot y^{\frac{2}{3}}\,dy$$

$$=\int_0^8 40\pi\cdot y^{\frac{2}{3}}-4\pi\cdot y^{\frac{5}{3}}\,y$$

$$=40\pi\cdot\frac{3}{5}\cdot y^{\frac{5}{3}}-4\pi\cdot\frac{3}{8}\cdot y^{\frac{8}{3}}\Big|_0^8$$

$$=24\pi\cdot 8^{\frac{5}{3}}-\frac{3}{2}\pi\cdot 8^{\frac{8}{3}}$$

$$=24\pi\cdot 2^5-\frac{3}{2}\pi\cdot 2^8$$

$$=24\pi\cdot 32-3\cdot\pi\cdot 128$$

$$=384\pi\cdot \text{lb-ft}.$$

2. Evaluate the following limits.

(a) (4 points) $\lim_{x\to 0} \frac{x^2}{\ln(\sec(3x))}$ $\left(\frac{0}{\ln 1} = \frac{0}{0} \text{ case}\right)$

$$= \lim_{x\to 0} \frac{2x}{\frac{1}{\sec(3x)} \cdot \tan(3x) \cdot \sec(3x) \cdot 3}$$

$$= \lim_{x\to 0} \frac{2x}{\tan(3x) \cdot 3} \quad \left(\frac{0}{0}\right)$$

$$= \lim_{x\to 0} \frac{2}{\sec^2(3x) \cdot 3 \cdot 3} = \frac{2}{\sec^2 0 \cdot 9} = \boxed{\frac{2}{9}}$$

(b) (5 points) $\lim_{x\to\infty} \frac{\ln x}{\log_5(2x)}$ $\left(\frac{\infty}{\infty}\right)$

$$= \lim_{x\to\infty} \frac{\frac{1}{x}}{\frac{1}{\ln 5} \cdot \frac{1}{2x} \cdot 2}$$

$$= \lim_{x\to\infty} \ln 5 = \boxed{\ln 5}$$

(c) (5 points) $\lim_{x\to\infty} (2x)^{3/x}$ $= \lim_{x\to\infty} e^{\ln (2x)^{\frac{3}{x}}} = e^{0} = \boxed{1}$

$$\lim_{x\to\infty} \ln(2x)^{\frac{3}{x}} = \lim_{x\to\infty} \frac{3\ln(2x)}{x} \quad \left(\frac{\infty}{\infty}\right)$$

$$= \lim_{x\to\infty} \frac{3 \cdot \frac{1}{2x} \cdot 2}{1} = \lim_{x\to\infty} \frac{3}{x} = 0$$

3. Find the derivative for each of the following functions.

   (a) (4 points) $y = 2\sin(6xe^{3x})$

$$y' = 2 \cdot \cos(6x \cdot e^{3x}) \cdot (6x \cdot e^{3x})'$$

$$= \boxed{2 \cdot \cos(6x \cdot e^{3x})(6 \cdot e^{3x} + 6x \cdot e^{3x} \cdot 3)}$$

   (b) (5 points) $g(x) = 7^{\ln(x^2-3x+1)}$

$$g'(x) = \ln 7 \cdot 7^{\ln(x^2-3x+1)} \cdot \frac{1}{x^2-3x+1} \cdot (2x-3).$$

   (c) (5 points) $y(x) = x^{7\sin(2x)}$

$$\ln y = 7\sin(2x) \cdot \ln x.$$

$$\frac{1}{y} \cdot y' = 7 \cdot \cos(2x) \cdot 2 \cdot \ln x + 7 \cdot \sin(2x) \cdot \frac{1}{x}.$$

$$y' = y \cdot \left[14\cos(2x) \cdot \ln x + \frac{7}{x}\sin(2x)\right].$$

4. Evaluate the following integrals.

(a) (7 points) $\int \sqrt{1-x^2}\,dx$ with $|x| \le 1$.

$x = \sin\theta$, $dx = \cos\theta\, d\theta$

$= \int \sqrt{1-\sin^2\theta}\,\cos\theta\, d\theta$

$= \int \cos\theta \cdot \cos\theta\, d\theta$

$= \int \frac{1+\cos 2\theta}{2}\, d\theta$

$= \frac{1}{2}\theta + \frac{1}{2}\cdot\frac{1}{2}\sin 2\theta + C$

$= \frac{1}{2}\theta + \frac{1}{4}\cdot 2\sin\theta\cdot\cos\theta + C$

$= \boxed{\frac{1}{2}\sin^{-1}x + \frac{1}{2}\cdot x\cdot\sqrt{1-x^2} + C}$

(b) (7 points) $\int \frac{x^2+4}{x^3+x^2}\,dx$

$\frac{x^2+4}{x^2(x+1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+1}$

$x^2+4 = A\cdot x\cdot(x+1) + B(x+1) + Cx^2$

$x^2+4 = (A+C)x^2 + (A+B)x + B$

$\begin{cases} A+C=1 \\ A+B=0 \\ B=4 \end{cases} \Rightarrow \begin{cases} A=-4 \\ B=4 \\ C=5 \end{cases}$

$\int \frac{x^2+4}{x^3+x^2}\,dx = \int \frac{-4}{x} + \frac{4}{x^2} + \frac{5}{x+1}\,dx = \boxed{-4\ln|x| + \frac{-4}{x} + 5\ln|x+1| + C}$

5. Solve the following initial value problems.

(a) (7 points) $\frac{dy}{dt} = te^t, \quad y(0) = 1$

$y = \int t \cdot e^t dt$ (IBP: $u = t$, $du = dt$, $dv = e^t dt$, $v = e^t$)

$= t \cdot e^t - \int e^t dt$

$= t \cdot e^t - e^t + C$.

$t=0$, $y=1 \Rightarrow 1 = 0 - e^0 + C \Rightarrow C = 2$

$\Rightarrow \boxed{y(t) = te^t - e^t + 2}$

(b) (7 points) $y''(x) = 12e^{2x} + 2, \quad y(0) = 4, \quad y'(0) = 5$

$y'(x) = \int 12 \cdot e^{2x} + 2 \; dx$

$= 6 \cdot e^{2x} + 2x + C_1 = 6 \cdot e^{2x} + 2x - 1$.

$y'(0) = 6 \cdot e^0 + 0 + C_1 = 5 \Rightarrow C_1 = -1$.

$y(x) = \int y' dx = \int 6 e^{2x} + 2x - 1 \; dx$

$= 3e^{2x} + x^2 - x + C_2 = \boxed{3e^{2x} + x^2 - x + 1}$

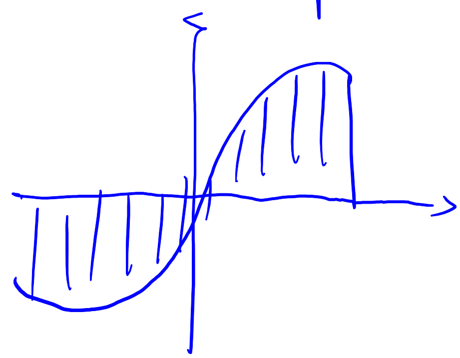
$4 = y(0) = 3 \cdot e^0 + 0 - 0 + C_2 \Rightarrow C_2 = 1$

Multiple Choice. Circle the single best answer. No work needed. No partial credit available.

6. (4 points) Mark the correct answer for the integral given by $\int_{-a}^{a} \sin(\sin(x^3))dx$.

   A. 0
   B. ∞
   C. $2a$
   D. a
   E. None of the above.

odd function



7. (4 points) Compute the improper integral and mark the correct answer $\int_3^6 \frac{1}{\sqrt{x-3}}dx$

   A. 0
   B. ∞
   C. $2\sqrt{3}$
   D. $\sqrt{3}$
   E. None of the above.

$= \lim_{t\to 3^+} \int_t^6 \frac{1}{\sqrt{x-3}} dx$ $\qquad \frac{1}{\sqrt{x-3}} = (x-3)^{-\frac{1}{2}}$

$= \lim_{t\to 3^+} 2\cdot(x-3)^{\frac{1}{2}} \Big|_t^6$

$= \lim_{t\to 3^+} 2(6-3)^{\frac{1}{2}} - 2(t-3)^{\frac{1}{2}} = \boxed{2\sqrt{3} - 0}$

8. (4 points) Compute the $\frac{d}{dx}\ln(\sinh(7x))$ and mark the correct answer

   A. $7\frac{\cosh x}{\sinh x}$
   B. $\frac{\sinh x}{\cosh x}$
   C. $\frac{\cos x}{\sinh x}$
   D. 0
   E. None of the above.

$= \frac{1}{\sinh(7x)} \cdot \cosh(7x) \cdot 7$

9. (4 points) Simplify as much as possible the expression $\dfrac{e^{3\ln(x-1)^2}}{\ln e^{(x-1)^4}}$

A. $(x-1)$

B. $(x-1)^{3/2}$

C. $(x-1)^2$ ✓

D. $(x-1)^4$

E. $(x-1)^6$

$$\frac{\cancel{e}^{\cancel{\ln}[(x-1)^2]^3}}{\cancel{\ln e}^{(x-1)^4}} = \frac{(x-1)^6}{(x-1)^4} = (x-1)^2$$

10. (4 points) If $f(3) = 2$, $f'(3) = 7$, then $(f^{-1})'(2) =$

A. $1/2$

B. $1/3$

C. $1/7$ ✓

D. $-1/2$

E. None of the above

$\Updownarrow$

$f^{-1}(2) = 3$

$$\frac{1}{f'(f^{-1}(2))} = \frac{1}{f'(3)} = \frac{1}{7}.$$

11. (4 points) $f(x) = \dfrac{6}{(x-1)(x+1)}$ can be written in the form

A. $\dfrac{3}{x-1} + \dfrac{3}{x+1}$

B. $\dfrac{-3}{x-1} + \dfrac{3}{x+1}$

C. $\dfrac{3}{x-1} + \dfrac{-3}{x+1}$ ✓

D. $\dfrac{-3}{x-1} + \dfrac{-3}{x+1}$

E. None of the above

$$= \frac{A}{x-1} + \frac{B}{x+1} = \frac{3}{x-1} + \frac{-3}{x+1}.$$

$6 = A\cdot(x+1) + B\cdot(x-1)$

$x=1 \Rightarrow 6 = 2A \quad A = 3$

$x=-1 \Rightarrow 6 = -2B, \quad B = -3$

12. (4 points) It took 100 J of work to stretch a spring from its natural length of 3 m to a length of 8 m. Find the spring force constant

    A. $k = 80/11$

    B. $k = 40/11$

    C. $k = 8$

    D. $k = 4$

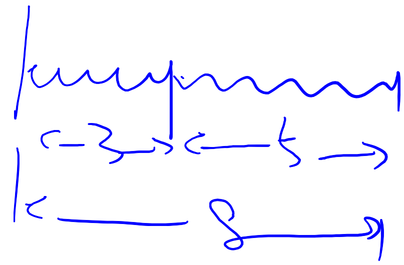
    E. None of the above

$F = k \cdot x.$

$W = \int_0^5 kx\,dx$

$= \frac{1}{2} k \cdot x^2 \Big|_0^5$

$= \frac{1}{2} \cdot k \cdot 25 - 0 = 100 \Rightarrow k = \frac{2 \cdot 100}{25} = 8$



13. (4 points) Evaluate $\int \frac{1}{\sqrt{1-x^2}}\,dx$ with ~~$|x| < 1$~~

    A. $\sin^{-1}(x) + C$

    B. $\cos^{-1}(x) + C$

    C. $\tan^{-1}(x) + C$

    D. $\cot^{-1}(x) + C$

    E. None of the above

$= \sin^{-1} x.$

14. (4 points) $g(t) = e^{t^2+2\sqrt{t}}$

    A. $e^{t^2+2\sqrt{t}} \cdot (2t + 1/\sqrt{t})$

    B. $e^{t^2+2\sqrt{t}}$

    C. $e^{2t+1/\sqrt{t}} \cdot (2t + 1/\sqrt{t})$

    D. $e^{2t+1/\sqrt{t}}$

    E. None of the above

$g'(t) = e^{t^2+2\sqrt{t}} \cdot \left(2t + 2 \cdot \frac{1}{2} t^{-\frac{1}{2}}\right)$

$= e^{t^2+2\sqrt{t}} \cdot \left(2t + \frac{1}{\sqrt{t}}\right)$

Congratulations you are now done with the exam!
Go back and check your solutions for accuracy and clarity. Make sure your final answers are $\boxed{\text{BOXED}}$.

When you are completely happy with your work please bring your exam to the front to be handed in. **Please have your MSU student ID ready** so that is can be checked.

DO NOT WRITE BELOW THIS LINE.

Page	Points	Score
2	14	
3	14	
4	14	
5	14	
6	14	
7	12	
8	12	
9	12	
Total:	106	

No more than 100 points may be earned on the exam.

FORMULA SHEET

Integrals

- **Volume**: Suppose $A(x)$ is the cross-sectional area of the solid S perpendicular to the x-axis, then the volume of S is given by

$$V = \int_a^b A(x)\ dx$$

- **Work**: Suppose $f(x)$ is a force function. The work in moving an object from a to b is given by:

$$W = \int_a^b f(x)\ dx$$

- $\displaystyle\int \frac{1}{x}\ dx = \ln|x| + C$
- $\displaystyle\int \tan x\ dx = \ln|\sec x| + C$
- $\displaystyle\int \sec x\ dx = \ln|\sec x + \tan x| + C$
- $\displaystyle\int a^x\ dx = \frac{a^x}{\ln a} + C$ for $a \neq 1$
- **Integration by Parts**:

$$\int u\ dv = uv - \int v\ du$$

Derivatives

- $\dfrac{d}{dx}(\sinh x) = \cosh x$ $\quad\dfrac{d}{dx}(\cosh x) = \sinh x$
- Inverse Trigonometric Functions:

$$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}(\csc^{-1} x) = \frac{-1}{x\sqrt{x^2-1}}$$

$$\frac{d}{dx}(\cos^{-1} x) = \frac{-1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}(\sec^{-1} x) = \frac{1}{x\sqrt{x^2-1}}$$

$$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2} \qquad \frac{d}{dx}(\cot^{-1} x) = \frac{-1}{1+x^2}$$

- If f is a one-to-one differentiable function with inverse function f^{-1} and $f'(f^{-1}(a)) \neq 0$, then the inverse function is differentiable at a and

$$(f^{-1})'(a) = \frac{1}{f'(f^{-1}(a))}$$

Hyperbolic and Trig Identities

- Hyperbolic Functions

$$\sinh(x) = \frac{e^x - e^{-x}}{2} \qquad \operatorname{csch}(x) = \frac{1}{\sinh x}$$

$$\cosh(x) = \frac{e^x + e^{-x}}{2} \qquad \operatorname{sech}(x) = \frac{1}{\cosh x}$$

$$\tanh(x) = \frac{\sinh x}{\cosh x} \qquad \coth(x) = \frac{\cosh x}{\sinh x}$$

- $\cosh^2 x - \sinh^2 x = 1$
- $\cos^2 x + \sin^2 x = 1$
- $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$
- $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$
- $\sin(2x) = 2\sin x \cos x$
- $\sin A \cos B = \frac{1}{2}[\sin(A-B) + \sin(A+B)]$
- $\sin A \sin B = \frac{1}{2}[\cos(A-B) - \cos(A+B)]$