

Name: _____

Section: _____ Recitation Instructor: _____

READ THE FOLLOWING INSTRUCTIONS.

- **Do not open your exam until told to do so.**
- No calculators, cell phones or any other electronic devices can be used on this exam.
- Clear your desk of everything excepts pens, pencils and erasers.
- If you need scratch paper, use the back of the previous page.
- Without fully opening the exam, check that you have pages 1 through 11.
- Fill in your name, etc. on this first page.
- **Show all your work.** Write your answers clearly! Include enough steps for the grader to be able to follow your work. Don't skip limits or equal signs, etc. Include words to clarify your reasoning.
- Do first all of the problems you know how to do immediately. Do not spend too much time on any particular problem. Return to difficult problems later.
- If you have any questions please raise your hand and a proctor will come to you.
- There is no talking allowed during the exam.
- You will be given exactly 90 minutes for this exam.
- Remove and utilize the formula sheet provided to you at the end of this exam.
- This is a practice exam. The actual exam may differ significantly from this practice exam because there are many varieties of problems that can test each concept.

I have read and understand the above instructions: _____

SIGNATURE

2. Evaluate the following limits.

(a) (4 points) $\lim_{x \rightarrow 0} \frac{x^2}{\ln(\sec(3x))}$

(b) (5 points) $\lim_{x \rightarrow \infty} \frac{\ln x}{\log_5(2x)}$

(c) (5 points) $\lim_{x \rightarrow \infty} (2x)^{3/x}$

3. Find the derivative for each of the following functions.

(a) (4 points) $y = 2 \sin(6xe^{3x})$

(b) (5 points) $g(x) = 7^{\ln(x^2-3x+1)}$

(c) (5 points) $y(x) = x^{7 \sin(2x)}$

4. Evaluate the following integrals.

(a) (7 points) $\int \sqrt{1-x^2} dx$ with $|x| \leq 1$.

(b) (7 points) $\int \frac{x^2 + 4}{x^3 + x^2} dx$

5. Solve the following initial value problems.

(a) (7 points) $\frac{dy}{dt} = te^t, \quad y(0) = 1$

(b) (7 points) $y''(x) = 12e^{2x} + 2, \quad y(0) = 4, \quad y'(0) = 5$

Multiple Choice. Circle the single best answer. No work needed. No partial credit available.

6. (4 points) Mark the correct answer for the integral given by $\int_{-a}^a \sin(\sin(x^3))dx$.

- A. 0
- B. ∞
- C. $2a$
- D. a
- E. None of the above.

7. (4 points) Compute the improper integral and mark the correct answer $\int_3^6 \frac{1}{\sqrt{x-3}}dx$

- A. 0
- B. ∞
- C. $2\sqrt{3}$
- D. $\sqrt{3}$
- E. None of the above.

8. (4 points) Compute the $\frac{d}{dx}\ln(\sinh(7x))$ and mark the correct answer

- A. $7\frac{\cosh x}{\sinh x}$
- B. $\frac{\sinh x}{\cosh x}$
- C. $\frac{\cos x}{\sinh x}$
- D. 0
- E. None of the above.

9. (4 points) Simplify as much as possible the expression $\frac{e^{3\ln(x-1)^2}}{\ln e^{(x-1)^4}}$

- A. $(x - 1)$
- B. $(x - 1)^{3/2}$
- C. $(x - 1)^2$
- D. $(x - 1)^4$
- E. $(x - 1)^6$

10. (4 points) If $f(3) = 2$, $f'(3) = 7$, then $(f^{-1})'(2) =$

- A. $1/2$
- B. $1/3$
- C. $1/7$
- D. $-1/2$
- E. None of the above

11. (4 points) $f(x) = \frac{6}{(x-1)(x+1)}$ can be written in the form

- A. $\frac{3}{x-1} + \frac{3}{x+1}$
- B. $\frac{-3}{x-1} + \frac{3}{x+1}$
- C. $\frac{3}{x-1} + \frac{-3}{x+1}$
- D. $\frac{-3}{x-1} + \frac{-3}{x+1}$
- E. None of the above

12. (4 points) It took 100 J of work to stretch a spring from its natural length of 3 m to a length of 8 m. Find the spring force constant

A. $k = 80/11$
B. $k = 40/11$
C. $k = 8$
D. $k = 4$
E. None of the above

13. (4 points) Evaluate $\int \frac{1}{\sqrt{1-x^2}} dx$ with $|x| < 1$.

A. $\sin^{-1}(x) + C$
B. $\cos^{-1}(x) + C$
C. $\tan^{-1}(x) + C$
D. $\cot^{-1}(x) + C$
E. None of the above

14. (4 points) $g(t) = e^{t^2+2\sqrt{t}}$

A. $e^{t^2+2\sqrt{t}} \cdot (2t + 1/\sqrt{t})$
B. $e^{t^2+2\sqrt{t}}$
C. $e^{2t+1/\sqrt{t}} \cdot (2t + 1/\sqrt{t})$
D. $e^{2t+1/\sqrt{t}}$
E. None of the above

Congratulations you are now done with the exam!

Go back and check your solutions for accuracy and clarity. Make sure your final answers are **BOXED**.

When you are completely happy with your work please bring your exam to the front to be handed in.

Please have your MSU student ID ready so that it can be checked.

DO NOT WRITE BELOW THIS LINE.

Page	Points	Score
2	14	
3	14	
4	14	
5	14	
6	14	
7	12	
8	12	
9	12	
Total:	106	

No more than 100 points may be earned on the exam.

FORMULA SHEET

Integrals

- **Volume:** Suppose $A(x)$ is the cross-sectional area of the solid S perpendicular to the x -axis, then the volume of S is given by

$$V = \int_a^b A(x) dx$$

- **Work:** Suppose $f(x)$ is a force function. The work in moving an object from a to b is given by:

$$W = \int_a^b f(x) dx$$

- $\int \frac{1}{x} dx = \ln|x| + C$
- $\int \tan x dx = \ln|\sec x| + C$
- $\int \sec x dx = \ln|\sec x + \tan x| + C$
- $\int a^x dx = \frac{a^x}{\ln a} + C$ for $a \neq 1$
- **Integration by Parts:**

$$\int u dv = uv - \int v du$$

Derivatives

$$\bullet \frac{d}{dx}(\sinh x) = \cosh x \quad \frac{d}{dx}(\cosh x) = \sinh x$$

- Inverse Trigonometric Functions:

$$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}} \quad \frac{d}{dx}(\csc^{-1} x) = \frac{-1}{x\sqrt{x^2-1}}$$

$$\frac{d}{dx}(\cos^{-1} x) = \frac{-1}{\sqrt{1-x^2}} \quad \frac{d}{dx}(\sec^{-1} x) = \frac{1}{x\sqrt{x^2-1}}$$

$$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2} \quad \frac{d}{dx}(\cot^{-1} x) = \frac{-1}{1+x^2}$$

- If f is a one-to-one differentiable function with inverse function f^{-1} and $f'(f^{-1}(a)) \neq 0$, then the inverse function is differentiable at a and

$$(f^{-1})'(a) = \frac{1}{f'(f^{-1}(a))}$$

Hyperbolic and Trig Identities

- Hyperbolic Functions

$$\sinh(x) = \frac{e^x - e^{-x}}{2} \quad \operatorname{csch}(x) = \frac{1}{\sinh x}$$

$$\cosh(x) = \frac{e^x + e^{-x}}{2} \quad \operatorname{sech}(x) = \frac{1}{\cosh x}$$

$$\tanh(x) = \frac{\sinh x}{\cosh x} \quad \operatorname{coth}(x) = \frac{\cosh x}{\sinh x}$$

- $\cosh^2 x - \sinh^2 x = 1$
- $\cos^2 x + \sin^2 x = 1$
- $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$
- $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$
- $\sin(2x) = 2 \sin x \cos x$
- $\sin A \cos B = \frac{1}{2}[\sin(A - B) + \sin(A + B)]$
- $\sin A \sin B = \frac{1}{2}[\cos(A - B) - \cos(A + B)]$