Name: ______________________ **ID:** ______________

Clear your desk of everything except pens, pencils and erasers. Show all work clearly and in order. No notes, phones and calculators. You have 10 minutes to finish these two problems for 10 points. (**Formula Sheet is on the back.**)

1. (5 points) Write the following POWER SERIES in sigma-notation as $\sum_{n=0}^{\infty} c_n(x-a)^n$, giving an explicit formula for c_n. Find its RADIUS of CONVERGENCE.

$$3 - \frac{3}{2}x + \frac{3}{4}x^2 - \frac{3}{8}x^3 + \frac{3}{16}x^4 - \cdots = \sum_{n=0}^{\infty} 3 \cdot \left(-\frac{1}{2}\right)^n \cdot x^n$$

$c_0 = 3$, $c_1 = -\frac{3}{2}$, $c_2 = \frac{3}{4}$, $c_3 = -\frac{3}{8}$, $c_4 = \frac{3}{16}$, ...

$c_n = 3 \cdot \left(-\frac{1}{2}\right)^n$, $n = 0, 1, 2, 3, 4, \ldots$

$a_n = 3 \cdot \left(-\frac{1}{2}\right)^n \cdot x^n$. Apply Ratio Test to a_n

$$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \frac{\left|3 \cdot \left(-\frac{1}{2}\right)^{n+1} \cdot x^{n+1}\right|}{\left|3 \cdot \left(-\frac{1}{2}\right)^n \cdot x^n\right|} = \lim_{n\to\infty} \frac{\left|\frac{1}{2^{n+1}} \cdot x^{n+1}\right|}{\left|\frac{1}{2^n} \cdot x^n\right|} = \lim_{n\to\infty} \frac{|x|}{2} = \frac{|x|}{2} < 1$$

$\Rightarrow |x| < 2$. Therefore, the radius of convergence is $\boxed{R=2}$

2. (5 points) Find the first FOUR NON-ZERO terms in the power series representation of $f(x) = \frac{x}{x+2}$ and find its RADIUS of CONVERGENCE.

$$\frac{x}{x+2} = \frac{x}{2\left[1+\frac{x}{2}\right]} = \frac{x}{2} \cdot \frac{1}{1-\left(-\frac{x}{2}\right)} = \frac{x}{2} \cdot \sum_{n=0}^{\infty} \left(-\frac{x}{2}\right)^n$$

$$= \frac{x}{2} \cdot \left[1 - \frac{x}{2} + \frac{x^2}{4} - \frac{x^3}{8} + \cdots\right]$$

$$= \boxed{\frac{x}{2} - \frac{x^2}{4} + \frac{x^3}{8} - \frac{x^4}{16}} + \cdots$$

Radius of Convergence:

$\left|-\frac{x}{2}\right| < 1 \Rightarrow |x| < 2$. $\boxed{R=2}$