Name: ______________________ **ID:** ______________

Clear your desk of everything except pens, pencils and erasers. Show all work clearly and in order. No notes, phones and calculators. You have 10 minutes to finish these two problems for 10 points.

1. (4 points) Evaluate the following integral:

$$\int \cos^3 t \cdot \sin^\pi t \, dt$$

Hint: $\cos^3 t$ → odd power, substitute "the other"

$u = \sin t$, $du = \cos t \cdot dt$ $\Rightarrow \frac{du}{\cos t} = dt$

$$\xrightarrow{\text{substitute}} \int \cos^3 t \cdot \sin^\pi t \, dt = \int \cos^3 t \cdot u^\pi \cdot \frac{du}{\cos t} = \int \cos^2 t \cdot u^\pi \cdot du$$

Hint: $\cos^2 t = 1 - \sin^2 t = 1 - u^2$

$$= \int (1-u^2) \cdot u^\pi \cdot du$$

$$= \int u^\pi - u^{2+\pi} \cdot du$$

Hint: $\int x^n dx = \frac{1}{n+1} \cdot x^{n+1}$, $n \neq -1$

π is just a constant.

$$= \frac{1}{\pi+1} \cdot u^{\pi+1} - \frac{1}{3+\pi} \cdot u^{3+\pi} + C$$

$$= \boxed{\frac{1}{\pi+1} \cdot \sin^{\pi+1} x - \frac{1}{3+\pi} \cdot \sin^{3+\pi} x + C}$$

2. (6 points) Evaluate the integral, your answer should NOT contain any TRIGONOMETRIC FUNCTIONS:

$$\int \frac{x}{\sqrt{x^2+4}} \, dx$$

Trig-Sub: $x = 2\tan\theta$

$dx = 2\sec^2\theta \cdot d\theta$

$$\int \frac{x}{\sqrt{x^2+4}} \cdot dx$$

$$= \int \frac{2\tan\theta \cdot}{\sqrt{(2\tan\theta)^2+4}} 2\sec^2\theta \, d\theta$$

$$= \int \frac{2\tan\theta}{2\sec\theta} 2\sec^2\theta$$

$$= \int \tan\theta \cdot \sec\theta \cdot d\theta$$

$$= \sec\theta + C$$

$$= \boxed{\frac{\sqrt{x^2+4}}{2} + C}$$

Hint:

$$\sqrt{(2\tan\theta)^2+4}$$

$$= \sqrt{4\tan^2\theta + 4}$$

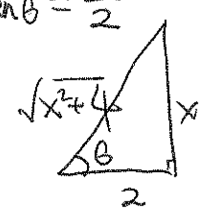
$$= \sqrt{4(\tan^2\theta + 1)}$$

$$= \sqrt{4 \cdot \sec^2\theta}$$

$$= 2\sec\theta$$

Hint: Trig-Id: $\tan^2\theta + 1 = \sec^2\theta$

Solve Triangle. $x = 2\tan\theta \Rightarrow \tan\theta = \frac{x}{2}$



$\Rightarrow \sec\theta = \frac{\sqrt{x^2+4}}{2}$

Remark: This problem is also solvable via u-sub $u = x^2+4$ (which is easier).

IT IS A GOOD PRACTICE FOR TRIG-SUB. IT IS STRONGLY RECOMMENDED TO PRACTICE IT VIA TRIG-SUB AS SHOWN on the left.