Name: _____ ID: _____

Clear your desk of everything except pens, pencils and erasers. Show all work clearly and in order. No notes, phones and calculators. You have 10 minutes to finish these two problems for 10 points.

1. (5 points) Evaluate the following integral:

$$\int \frac{\cosh\left(\tan^{-1}(2x)\right)}{1+4x^2}\mathrm{d}x$$

Hint:

$$\frac{\mathrm{d}}{\mathrm{d}x}(\cosh x) = \sinh x, \quad \frac{\mathrm{d}}{\mathrm{d}x}\tan^{-1} x = \frac{1}{1+x^2}$$

u-sub: $u = \tan^{-1}(2x)$, $du = \frac{1}{1+(2x)^2}\cdot 2\cdot dx$ ← Remark: Although the formula is given, you still have to be familiar with CHAIN RULE to compute du correctly

$\Rightarrow \frac{du}{2} = \frac{1}{1+4x^2}dx$.

$\int \frac{\cosh(\tan^{-1}(2x))}{1+4x^2}dx$

$= \int \cosh(u)\cdot\frac{1}{2}du$

$= \frac{1}{2}\cdot\sinh(u)+C$

$= \boxed{\frac{1}{2}\cdot\sinh(\tan^{-1}(2x))+C}$

2. (5 points) Evaluate the following limit:

$$\lim_{x\to 0}\frac{8x^2}{\cos x - 1}$$

(Remark: $\frac{0}{0}$ type since $\cos 0 = 1$.)

$\lim_{x\to 0}\frac{8x^2}{\cos x-1} \overset{\text{l'Hop}}{=} \lim_{x\to 0}\frac{(8x^2)'}{(\cos x-1)'}$

$= \lim_{x\to 0}\frac{8\cdot 2x}{-\sin x}$ ← Caution: $-\sin x$ NOT $\sin x$. Pay attention the negative sign.

$\overset{\text{l'Hop}}{=} \lim_{x\to 0}\frac{16}{-\cos x}$ ← $(\frac{0}{0})$.

$\overset{\text{Now plug in } x=0}{=} \frac{16}{-\cos 0} = \frac{16}{-1} = \boxed{-16}$.