Name: _____ **ID:** _____

Clear your desk of everything except pens, pencils and erasers. Show all work clearly and in order. No notes, phones and calculators. You have 15 minutes to finish these two problems for 10 points.

1. A vertical right cylindrical tank has **height 2** ft and **radius 1** ft. It is full of soda **weighing** 60 **lbs**/ft^3. How much work does it take to pump all of the soda from a tank to an outlet which is at **the level of the top of the tank**.

   (a) (4 points) Set up the integral for the work.

$$\begin{aligned} W &= \int_a^b \sigma s(y) A(y) \mathrm{d}y \\ &\qquad \sigma = 60,\ s(y) = 2 - y,\ A(y) = \pi \cdot 1^2 = \pi, a = 0, b = 2 \\ &= \int_0^2 60(2-y)\pi \mathrm{d}y \\ &= 60\pi \int_0^2 (2-y)\mathrm{d}y \end{aligned}$$

   (b) (2 points) Evaluate the integral.

$$\begin{aligned} W &= 60\pi \int_0^2 (2-y)\mathrm{d}y \\ &= 60\pi(2y - \frac{1}{2}y^2) \mid_0^2 \\ &= 60\pi(2 \times 2 - \frac{1}{2}2^2) \\ &= 120\pi \quad \text{ft-lbs} \end{aligned}$$

2. (4 points) Suppose that the differentiable function $y = f(x)$ has an inverse. The graph of f **passes through the origin with slope** 4, i.e., $f(0) = 0$ and $f'(0) = 4$. Find the slope of the graph of f^{-1} **at the origin**.

$$\textbf{Hint}: (f^{-1})'(a) = \frac{1}{f'\big(f^{-1}(a)\big)}$$

   Solution: The slope of the graph of f^{-1} at the origin equals the derivative of f^{-1} at $x = 0$, i.e.,

$$\text{slope} = (f^{-1})'(0) = \frac{1}{f'\big(f^{-1}(0)\big)} = \frac{1}{f'(0)} = \frac{1}{4}$$