Name: ______________________ **ID:** ______________

Clear your desk of everything except pens, pencils and erasers. Show all work clearly and in order. No notes, phones and calculators. You have 10 minutes to finish these two problems for 10 points.

1. (6 points) Evaluate the following limits. Hint: Apply L'Hospital's Rule to part (a) and use part (a) to draw the conclusion for part (b) directly.

(a)

$$\lim_{x\to 0}\frac{\sin x}{x} \overset{\text{l'Hosp}}{=} \lim_{x\to 0}\frac{\cos x}{1} = \cos 0 = \boxed{1.}$$

(b) Consider $\frac{1}{3n^2}$ as x in (a), we have $\lim_{n\to\infty}\frac{1}{3n^2}=0$

Therefore, $\lim_{n\to\infty} n^2\cdot\sin(\frac{1}{3n^2}) = \lim_{n\to\infty}\frac{1}{3}\cdot 3n^2\cdot\sin(\frac{1}{3n^2})$

$= \lim_{n\to\infty}\frac{1}{3}\cdot\boxed{\frac{\sin(\frac{1}{3n^2})}{\frac{1}{3n^2}}} = \lim_{n\to\infty}\frac{1}{3}\cdot 1 = \boxed{\frac{1}{3}}$

$\sin(x)$

x

(b)

$$\lim_{n\to+\infty} n^2\cdot\sin(\frac{1}{3n^2})$$

(b) Direct l'Hospital method.

$\lim_{n} n^2\cdot\sin(\frac{1}{3n^2}) = \lim_{n\to\infty}\frac{\sin(\frac{1}{3n^2})}{\frac{1}{n^2}} \overset{\text{l'Hosp}}{=} \lim_{n}\frac{\cos(\frac{1}{3n^2})\cdot\cancel{\frac{-2}{3\cdot n^3}}}{\cancel{\frac{-2}{n^3}}} = \lim_{n\to\infty}\cos(\frac{1}{3n^2})\cdot\frac{1}{3} = \cos 0\cdot\frac{1}{3}$

$= \boxed{\frac{1}{3}}$

2. (4 points) Find the equation of the line tangent to the parametric curve

$$x(t) = \ln t,\quad y(t) = (t-1)^2 + 3t$$

at $t = 1$.

$t=1$, $x(1)=\ln 1=0$, $y(1)=(1-1)^2+3=3.$

$\frac{dx}{dt} = \frac{1}{t} = \frac{1}{1} = 1$, $\frac{dy}{dt} = 2(t-1)+3 = 2(1-1)+3 = 3.$

$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{3}{1} = 3$ slope;

tangent line: $y = 3 + 3\cdot(x-0) \Rightarrow \boxed{y=3+3x}$