Build your OWN!

Integrals

- **Volume**: Suppose $A(x)$ is the cross-sectional area of the solid S perpendicular to the x-axis, then the volume of S is given by

Rotating case:

$A(x)=\pi\cdot[f(x)-\text{AXIS}]^2$ $\qquad V=\int_a^b A(x)\,dx$

- **Work**: Suppose $f(x)$ is a force function. The work in moving an object from a to b is given by:

Water-Pumping:

$W=\int_a^b 6\cdot s(y)\cdot A(y)\,dy$ $\qquad W=\int_a^b f(x)\,dx$ $\qquad (\sqrt{x})'=\frac{1}{2\sqrt{x}}$, $\quad (\frac{1}{x})'=-\frac{1}{x^2}$

- $\int \frac{1}{x}\,dx = \ln|x| + C$, $\int x^n dx = \frac{1}{n+1}x^{n+1}+C$, $n\neq -1$
- $\int \tan x\,dx = \ln|\sec x| + C$, $(\tan x)'=\sec^2 x$, $(\cot x)'=-\csc^2 x$

  $(\sec x)'=\tan x\cdot\sec x$, $(\csc x)'=-\cot x\cdot\csc x$.
- $\int \sec x\,dx = \ln|\sec x+\tan x| + C$

  $(a^x)'=\ln a\cdot a^x$
- $\int a^x\,dx = \frac{a^x}{\ln a} + C$ $\quad$ for $a\neq 1$

  $a^x = e^{\ln a\cdot x}$.

  $a^{-x}=\frac{1}{a^x}$, $a^x=\frac{1}{a^{-x}}$.
- **Integration by Parts**:

$$\int u\,dv = uv - \int v\,du$$

$\int \underbrace{\text{Poly}}_{u} \times \underbrace{\exp/\sin x/\cos x \; dx}_{dv}$

$dv=e^{ax}dx \Rightarrow v=\int e^{ax}dx=\frac{1}{a}e^{ax}$

$dv=\sin(ax)dx \Rightarrow v=\int \sin(ax)dx=-\frac{1}{a}\cos(ax)$

$dv=\cos(ax)dx \Rightarrow v=\int \cos(ax)dx=\frac{1}{a}\sin(ax)$

- limits: $e^{+\infty}=+\infty$, $e^{-\infty}=0$, $\ln\infty=\infty$, $\ln 0^+=-\infty$

  $\tan^{-1}(\pm\infty)=\pm\frac{\pi}{2}$. $\qquad e^0=1$, $\ln 1=0$
- L'Hôpital Rule: $\lim_{x\to a}\frac{f(x)}{g(x)}=\lim_{x\to a}\frac{f'(x)}{g'(x)}$

  $\frac{\infty}{\infty}$, $\frac{0}{0}$, $\quad 0\cdot\infty=\frac{0}{\frac{1}{\infty}}=\frac{\infty}{\frac{1}{0}}$
- P.F.D. $\frac{*}{(x-a)(x-b)}=\frac{A}{x-a}+\frac{B}{x-b}$, $\frac{*}{(x-a)^2}=\frac{A}{x-a}+\frac{B}{(x-a)^2}$

  $\int\frac{1}{x-a}dx=\ln|x-a|$, $\int\frac{1}{(x-a)^2}dx=-\frac{1}{x-a}$.

Derivatives

- $\frac{d}{dx}(\sinh x)=\cosh x \qquad \frac{d}{dx}(\cosh x)=\sinh x$
- Inverse Trigonometric Functions: $\int\frac{1}{\sqrt{1-x^2}}dx=\sin^{-1}x$, $\int\frac{dx}{\sqrt{a^2-b^2x^2}}$, $\quad$ u-Sub: $bx=au$

$\boxed{\frac{d}{dx}(\sin^{-1}x)=\frac{1}{\sqrt{1-x^2}}}$ $\qquad \frac{d}{dx}(\csc^{-1}x)=\frac{-1}{x\sqrt{x^2-1}}$

$\frac{d}{dx}(\cos^{-1}x)=\frac{-1}{\sqrt{1-x^2}}$ $\qquad \frac{d}{dx}(\sec^{-1}x)=\frac{1}{x\sqrt{x^2-1}}$

$\boxed{\frac{d}{dx}(\tan^{-1}x)=\frac{1}{1+x^2}}$ $\qquad \frac{d}{dx}(\cot^{-1}x)=\frac{-1}{1+x^2}$

$\int\frac{1}{a^2+x^2}dx=\frac{1}{a}\tan^{-1}(\frac{x}{a})$

- If f is a one-to-one differentiable function with inverse function f^{-1} and $f'(f^{-1}(a))\neq 0$, then the inverse function is differentiable at a and

$$(f^{-1})'(a)=\frac{1}{f'(f^{-1}(a))}$$

Hyperbolic and Trig Identities

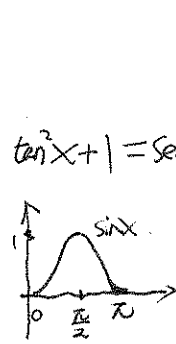
- Hyperbolic Functions

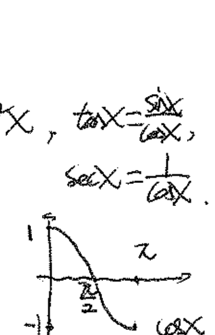
$$\sinh(x)=\frac{e^x-e^{-x}}{2} \qquad \operatorname{csch}(x)=\frac{1}{\sinh x}$$

$$\cosh(x)=\frac{e^x+e^{-x}}{2} \qquad \operatorname{sech}(x)=\frac{1}{\cosh x}$$

$$\tanh(x)=\frac{\sinh x}{\cosh x} \qquad \coth(x)=\frac{\cosh x}{\sinh x}$$

- $\cosh^2 x-\sinh^2 x=1$
- $\boxed{\cos^2 x+\sin^2 x=1}$ $\quad \tan^2 x+1=\sec^2 x$, $\tan x=\frac{\sin x}{\cos x}$, $\sec x=\frac{1}{\cos x}$.
- $\sin^2 x=\frac{1}{2}(1-\cos 2x)$
- $\cos^2 x=\frac{1}{2}(1+\cos 2x)$
- $\sin(2x)=2\sin x\cos x$





- $\sin A\cos B=\frac{1}{2}[\sin(A-B)+\sin(A+B)]$
- $\sin A\sin B=\frac{1}{2}[\cos(A-B)-\cos(A+B)]$
- $\cos A\cos B=\frac{1}{2}[\cos(A-B)+\cos(A+B)]$
- Trig-Sub: $\sqrt{a^2-b^2x^2}$ $\quad bx=a\sin\theta$

  $\sqrt{a^2+b^2x^2}$ $\quad bx=a\tan\theta$

  $\sqrt{b^2x^2-a^2}$ $\quad bx=a\sec\theta$.